



## Optimal, scalable forward models for gravity computations

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The application of forward models to compute a synthetic gravity signal is required to perform an inversion of the subsurface density structure. Numerous forward models exist, the most common of which is a direct summation approach employing an analytic expression to compute the gravity over a voxel. To permit the description of a detailed density structure, the domain is discretised into a set of voxels, each of which is assigned a density. The gravity at a single point  $s$ , is computed by evaluating the closed form expression for the gravity over each element, and summing the result.

The summation approach has several disadvantages. The operator  $F$  (i.e. the matrix) describing the forward model is completely dense. For inversions, the operator  $F^T \cdot F$  or  $F \cdot F^T$  is often required, and both these products will be dense if  $F$  is dense. Consequently the storage requirements are large for high spatial resolution 3D inversions if these products are required to be stored. Another drawback of the summation approach is the algorithmic scaling. If the domain contains  $N \times N \times N$  voxels, we are required to perform  $O(N^3)$  operations per gravity station. Hence, increasing the number of gravity stations, which are used as data in the inversion, increases the computational cost of the method.

An alternative to the summation based forward models is to solve the Poisson equation for the gravitational potential, and then compute the gravity field from the gradient of the potential. Using such a PDE based forward model has the advantage that the operator  $F$  is sparse and the computational cost is only very weakly dependent on the number of locations at which we wish to compute the gravity.

Here, we compare the accuracy and computational performance of two different approaches for solving the Poisson problem in 3D. The first approach consists of a finite element (FE) discretisation in which the resulting linear system is solved via a Krylov method, preconditioned with either geometric, or algebraic multigrid. The second approach uses the fast multipole method (FMM). Both methods are shown to be optimal, implying that the solution time scales linearly with respect to the number of elements used in the domain. Both approaches are implemented using PETSc (<http://www.mcs.anl.gov/petsc>) and are capable of running on massively parallel, distributed memory computers.

With respect to the FE method, we examine the accuracy and convergence of different approximations to the “zero at infinity” boundary condition including those used by Cai & Wang (2005) and Farquharson & Mosher (2009). Particular attention is given to the behaviour of the multilevel preconditioners when distorted, or stretched elements are used, and when different boundary condition approximations are employed. Maintaining the robustness of the preconditioner is essential to ensure this forward model remains efficient when used for high resolution 3D inversions. The accuracy, speed and optimality of the FMM are examined. Both the FE method and the FMM are compared with a classical, closed form summation forward model on the grounds of accuracy, CPU time and robustness.

## REFERENCES

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- Farquharson, C., Mosher, C., 2009. Three-dimensional modelling of gravity data using finite differences. *J. Appl. Geophys.* 68, 417–422.