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Compound solitons of Gardner equation with variable coefficients of cubic nonliarity

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The propagation of solitary waves in media with variable parameters is one of the classical issues in the theory of solitons. Soliton evolution in media with slowly (compared to the scale of solitary waves) varying parameters is the problem most studied by now. Quasi-stationarity of the wave process allows the solution to be regarded to be close to a stationary solitary wave with smoothly varying parameters, thus reducing description of solitary wave field transformation to a much simpler problem, namely, to description of dynamics of a finite number of independent soliton parameters. As much as the situation is regular and universal in a quasi-stationary case, so much diverse and specific is soliton field transformation when soliton scales are comparable with or exceed the scale îf medium changes. The process may tend to be quasi-linear in this case or may be accompanied either by cardinal soliton restructuring or by the birth of additional soliton waves. One can get an insight into the above mentioned nonstationary processes in the papers [1,2], where soliton transformation is studied numerically within the framework of the expanded KdV equation (Gardner equation) with variable parameters:

$$\Phi_t+\Phi\left(1-\mu(t)\Phi\right)\Phi_x+\Phi_{xxx}=0 \qquad (1)$$

Analysis of these works stimulated the authors to seek for an adequate approximate description of non-quasi-stationary evolution of solitons close to the limiting ones existing within the framework of the Gardner equation at $\mu>0$. The key feature in constructing an approximate solution is compound structure of such solitons. They have a plateau of arbitrary length S that is bounded by relatively narrow $(\sim\Phi\sqrt{\mu/6})^{-1}$ field kinks of different polarity that are also stationary solutions of Gardner equation at μ=const:

$$\Phi_{\pm}=\bar{\Phi}+\frac{\Phi_m}{2}\left[1\pm th\frac{1}{2}\sqrt{\frac{\mu}{6}}\cdot\Phi_m(x-V_m t)\right], \qquad (2)$$

where $\bar{\Phi}$ is arbitrary pedestal and $\Phi_m=1/\mu-2\bar{\Phi}, \frac{1}{6\mu}-\frac{\bar{\Phi}}{3}(1-\mu\bar{\Phi})=V_m$. Note that the difference between the velocity of the soliton close to a limiting one and the kink velocity is exponentially small. Below we consider the situation when at varying $\mu(t)$ the quasi-stationarity conditions are not fulfilled upon the whole for a relatively extended soliton but hold for the kinks forming the front and decay of the evolving quasi-soliton. The general solution describing dynamics of such a quasi-soliton conserves the compound structure: it has the form of kinks (2) with variable parameters in regions with rapidly changing field, and in regions with slowly changing field (between the front and the decay and behind the quasi-soliton decay) this solution is defined by equation (1) in the dispersion-free approximation:

$$x-x_0=\bar{\Phi}_(t-t_0)-\bar{\Phi}_2\int_t^t\mu(t')dt' \qquad (3)$$

The relationship between parameters x_0,t_0 is defined by the stationary relation (2) on the kink trajectory and gives a complete solution of the problem (1): front and decay parameters, form of the field describing the crest and field past the decay are found.

References

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