



## **A fast and accurate 3D finite-difference Eikonal solver using a perturbation method**

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Efficient calculation of first arrival travel times in gridded 3D volumes plays an important role in various areas of geophysics such as seismic tomography, earthquake location and also prestack Kirchhoff depth migration. To compute these traveltimes, eikonal equation solutions are widely used. They represent in many cases an accurate and cost effective way of computing traveltimes and these methods are in general much faster than ray tracing. Nevertheless, when compared with analytical solutions, present eikonal solvers exhibit some degree of traveltime error that may lead to poor image focusing in migration or inaccurate velocities estimated via tomographic inversion.

To overcome some of the limitations of present solvers, the algorithm proposed here consists in solving the eikonal equation using a perturbation method. This method offers a very high degree of accuracy in the solutions thus found, even when a very coarse sampling is used in the finite differencing step. In addition, this parsimony in the computational grid decreases significantly the number of operations to be conducted, thus allowing high speed of computation, and a significant savings in computer memory resources.

From the high-frequency approximation of the solution of the acoustic wave equation, the eikonal equation in cartesian coordinates is:  $|\nabla t|^2 = s^2$ . A fundamental property of this equation is that scaling slowness by a constant corresponds to scaling traveltime  $t$  by the same constant. This property allows us to derive the perturbation eikonal equation. We assume that  $s = s_0 \cdot \mu$  and  $t = t_0 \cdot \tau$  and also assume that  $s_0$  and  $t_0$  are also solution of the eikonal equation. After substitution, the equation to solve is the following:  $t_0^2 |\nabla \tau|^2 + 2T_0 \tau \nabla T_0 \nabla \tau + (\tau g - \mu^2) s_0 = 0$ .  $T_0$  and  $S_0$  are known from an analytical solution, the problem is finally the numerical evaluation of the correction  $\tau$ . This method produces traveltimes with very high accuracy. This eikonal solver produces almost exact results in homogeneous models. Our method accounts for transmission, diffraction and head waves and their direction of propagation. It can overcome the numerical instability of some FD methods in complex velocity models.