



## Bounds for effective Forchheimer coefficient in randomly heterogeneous porous media

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An understanding of the interplay between non-linear effects in porous media flow and domain heterogeneity is of great importance in several engineering and geological applications.

For a one-dimensional, statistically heterogeneous medium, we investigate non-linear flow caused by a uniform external pressure gradient, and described at the local scale by the Forchheimer equation; in the latter, the inertial effects are represented by adding to Darcy's law an additional term proportional to the fluid density and to the second power of the flow rate.

Since most experimental values of Forchheimer coefficient have been derived at the laboratory scale, a formula for its upscaling is much needed for the interpretation of field results in heterogeneous aquifers, when there is a reason to include nonlinear effects.

The permeability is considered a spatially homogeneous and correlated Gaussian random field with a given PDF, while the local Forchheimer coefficient  $\beta$  is related to the local permeability  $k$  value via the empirical inverse power-law correlation  $\beta = a/k^c$ , suggested in the literature on the basis of experimental data, with  $c$  an exponent in the range  $0 \div 2$ . Under the ergodic hypothesis, the effective permeability and Forchheimer coefficient are derived for two one-dimensional flow geometries: flow parallel to permeability variation (serial-type layers), and flow transversal to permeability variation (parallel-type layers); the expressions derived for the effective Forchheimer coefficient generalize those derived in the past for a discrete parameter variation. For the effective permeability, the classical results derived in the context of Darcy flow are recovered: the harmonic and the arithmetic mean respectively for the series and parallel arrangement.

For serial-type layers, an expression for the effective Forchheimer coefficient is derived in closed form; for parallel-type layers, an approximate analytical expression is derived under the hypothesis of small values of a dimensionless number  $Z$  including the pressure gradient. For higher values of  $Z$ , the effective Forchheimer coefficient is derived numerically. Upon comparing the results, the validity of the approximate numerical solution is assessed. The impact of the adoption of different shapes of the permeability PDF (lognormal or gamma) is also investigated.

As medium heterogeneity increases, the effective Forchheimer coefficient  $\beta_{eff}$  is found to increase for both geometries examined; for serial-type layers, it is also an increasing function of the exponent  $c$ ; the opposite for parallel-type layers. The effective Forchheimer coefficient is also moderately sensitive to the shape of the permeability PDF adopted.

Our results allow to evaluate the error made in the calculation of  $\beta_{eff}$  when the mean permeability value is inserted in the empirical correlation; the relative error increases with medium heterogeneity and can easily be in the range  $10 \div 100$ .

Results obtained for two limit geometries (serial- and parallel-type layers) in 1-D provide bounds for the evaluation of the effective Forchheimer coefficient in 2-D and 3-D flows.