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On the finding of the earth's surface topography with the help of toda-hierachy

Inna Stepanova
Institute of Physics of the Earth RAS,Moscow,Russia, e-mail: tet@ifz.ru

The determination of a function describing the Earth's surface topography on a generally irregular grid is important for the interpretation of gravity data, calculation of the corrections for the topography and construction of maps of plumb line deviations. Thus, we have the problem of describing a surface D, or more specifically, the problem of constructing an approximation of a surface D that will be used in the calculation of the function $g_3(\bar{r}) \equiv -\frac{\partial V}{\partial x_3}$ (the derivative of the potential with respect to the coordinate $x_3^{)}$. We can write $f_i = G(M_i) = -\frac{\partial V}{\partial x_3}(M_i) = \quad M_i = (x_1^{(i)}, x_2^{(i)}, x_3^{(i)}),\ 1 \le i \le N,$

where $r_i = (x_1^{(i)}, x_2^{(i)}, x_3^{(i)}),\ 1 \le i \le N$,are the observation points. If the model of the plane Earth is adopted, the surface D may be represented by a function of two variables ξ_1, ξ_2, parametrizing points on an equipotential surface.

Then, the correction for topographic masses (the gravity reduction) is calculated by the approximate formula

$$g_3(\bar{r}) = \gamma\sigma \iint\limits_{\Sigma} \left(\frac{1}{R(\xi|_{\Sigma} - \bar{r})} - \frac{1}{R(\xi_0 - \bar{r})} \right) d\xi_1 d\xi_2\,, \tag{1}$$

where, in the case of a detailed survey, the plane $\xi_0 = const$must be such that

(a) approximate formula (**??**) (ensuring a required accuracy) can be used for all $\bar{r}_i$, where $\Sigma$is a finite simply connected area in the plane (ξ_1, ξ_2) and

(b) for all $(\xi_1, \xi_2) \in \Sigma \quad \xi_3^0 - \xi_3(\xi_1, \xi_2) < 0$. (2)

The main computational procedure consists of the construction of an analytical approximation of the function $\Delta\xi_3(\xi_1, \xi_2) = \xi_3(\xi_1, \xi_2) - \xi_3^0$ (3)

From its values given on a set of points $(\xi_1^k, \xi_2^k), k = 1, ..., K$; this approximation should be valid on the entire integration domain. The analytical approximation $\Delta\xi_3(\xi_1, \xi_2)$can be constructed by the R-approximation method, which is a variant of the linear integral representation method based upon the Radon-transform.

The values $\Delta\xi_3(\xi_1, \xi_2)$used for the construction of the analytical approximation are given, first, immediately at the observation points; second, on a certain set of points of topographic maps on an adequate scale (for example, it is desirable to use topographic maps on a 1:10 000 scale in the case of a detailed gravity survey on a scale 1:25 000); third, on a certain set of characteristic points of the relief (these values must be measured during gravity surveys).

The method proposed in this topic for the Earth's topography description with the help of S and R-approximations is effective for solving various geophysical, geomorphological, an geodetic problems. Using this method, the values of the function describing the topography can be determined on an arbitrary grid. Therefore, the preliminary regularization of initial data, their ordering, and so on, are not required. A smoother topography is reconstructed to within a few centimeters, whereas mountainous topography is reconstructed less accurately (1-3 sm with a height contrast of about 1000 m).

When Σ(a finite simply connected area in the plane, as it is has been stressed before) is also unknown, we are to solve a nonlinear inverse problem of defining this area. It is well known, that a plane area with a sufficiently smooth boundary is completely determined by it's harmonic moments. The so called Darcy Law is valid in the filtration theory:

$$\vec{v} = -kgrad\,p.$$

Here $p$is the external pressure, k is the viscosity coefficient. We search for a conformal map of the exterior of a canonical area in the plane onto the exterior of our unknown domain. The external pressure is assumed to be known.

The conformal map described above is determined by means of dispersionless Toda hiearhieis. For the plane domains such as ellipses constructing the Toda hierarchies can be accomplished comparatively simply. This fact is proved in [*A.Zabrodin***.** Laplace growth problems, conformal maps and intergrable hierarhies November 12, 1999.

Wiegmann P.B., Zabrodin A., Conformal maps and integrable hierarchies, *Comm. Math. Phys.* **213** (2000), 523-538.]

The technique proposed in this contribution has been tested on some model samples.