



Laplace's Tidal Equations: Revisiting a 400 year old basic problem of GFD

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The classical problem of wavelike solutions of Laplace's Tidal Equations was studied for many decades but presently analytical solutions of these equations exist only on the equatorial β -plane and on a slowly rotating planet. The reason for this is the complex nature of the equations on a rotating sphere, which yields an intractable eigenvalue problem for the phase speed of the assumed zonally propagating wave solutions and their meridional structure. The advances in this problem are achieved by neglecting coefficients of the eigenvalue equation that arise only from the spherical geometry and retaining coefficients that have (less complex) counterparts in the planar problem. Our analytical results are obtained by transforming the eigenvalue problem to a form similar to that of Schrödinger equation of the harmonic oscillator of quantum mechanics. The essential approximation involved in this transformation can be formally justified in a baroclinic ocean (or atmosphere) and the accuracy of the analytical results is confirmed by comparing them to numerical solutions of the problem. The accuracy of our analytical estimates persists for mode numbers (i.e. meridional wavenumbers) of order up to 100.

The implication of the new approach is that in a baroclinic ocean on an aqua-planet (i.e. when no lateral coasts bound the thin ocean) only very high modes can be expected to exist at sufficiently high latitudes. The reason for this is that the eigenfunctions vanish, poleward of the "turning latitude" where the eigenvalue (i.e. energy level) becomes smaller than the monotonically increasing potential (as a function of latitude) of the eigenvalue problem. Such high modes are very hard to excite and their amplitude can not be sufficiently large for them to be observed.