



## A novel regularization scheme for electromagnetic inverse problems

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In the last years, there has been an ever growing interest in the development of electromagnetic non-destructive testing and evaluation techniques in civil engineering as well as in several other areas [1]. Radar-based methods, such as Ground Penetrating Radar (GPR), are now considered powerful tools for inspecting materials and structures. However, the inversion of scattered data is still considered a difficult task in many inverse-scattering-based reconstruction methods.

Formally, the inverse scattering problem can be modeled by an operator equation  $Ax = y$ , where  $A : X \rightarrow Y$  is an operator which maps the space  $X$  of the dielectric distributions of the investigation domain with the space  $Y$  of the scattered fields. In particular, by the knowledge of the measured scattered field  $y \in Y$  (i.e., the known data), the aim is to recover the related dielectric distribution  $x \in X$  (i.e., the unknown solution). Many classical resolution algorithms involving the linearization Gauss-Newton scheme allows one to approximate the solution  $x$  by means of the minimization of the least squares cost functional  $\Phi_2(x) = \|Ax - y\|_{L_2}$ , where  $\|\cdot\|_{L_2}$  is the classical Euclidean norm of the  $L^2$  functional Hilbert space [1]. Iterative regularization algorithms for the minimization of  $\Phi_2$  give in general oversmoothed solutions, which does not allow a well reconstruction of the discontinuities arising in natural dielectric distributions of different objects located in the domain of investigation.

In this work, we propose to solve the inverse scattering problem by means of the minimization of a different cost functional  $\Phi_p(x) = \|Ax - y\|_{L^p}$ , where  $\|\cdot\|_{L^p}$  is the norm of the  $L^p$  functional Banach space, defined as  $\|w\|_{L^p}^p = \int |w|^p dw$ . This way, the “size” of the residual  $r(x) = Ax - y$  is measured by means of the metric of the Banach space  $L^p$ , which emphasizes, for values of the constant  $1 < p < 2$ , the points where the residual is small with respect to the classical  $L^2$  norm. This choice gives rise to a substantial reduction of the over-smoothing in the restored solution  $x$ . Indeed the weight of the small values of the residual in the restoration process is enlarged, by allowing a further reduction of such a small values. A description of the developed inversion approach and some numerical results will be presented.

[1] M. Pastorino, Microwave Imaging. Hoboken, NJ: John Wiley & Sons, 2010.