



Fractal Objects in Geoscience Models

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Benoît Mandelbrot has greatly enriched our outlook on natural objects, by emphasizing the importance of irregularity in their contours and structure, and suggesting that much of this irregularity could be captured by scale-invariant descriptions. Nature, however, is parsimonious in the information it yields to direct observation, and scale-invariant cascades over many orders of magnitude are few and far between. Other speakers in this session and elsewhere report on some of the relevant and important situations where they do appear.

Fortunately, direct observation can and must be complemented by modelling, which permits certain types of extrapolation by indirect inference. This talk concerns several types of mathematical objects that have proven useful in modelling various phenomena in the geosciences and that do exhibit scale invariance or, to use the word introduced by Mandelbrot, fractality.

We first introduce scale invariance in some of the simple objects known since the late 19th and early 20th century, such as Pascal's triangle, the Cantor set and Peano curves. The talk proceeds to the late 20th century and the by now classical example of strange attractors of differentiable dynamical systems (DDS), in particular the Lorenz attractor. The main emphasis is on two somewhat more novel situations in which scale invariance plays a major role: (i) Boolean delay equations (BDEs), with their log-periodic solutions, and (ii) random dynamical systems (RDS), with their random attractors.