



Stability of a Stokes' wave train in the presence of uniform vorticity

Roland Thomas and Christian Kharif
IRPHE, France (kharif@irphe.univ-mrs.fr)

We study the stability of weakly nonlinear water waves in the presence of uniform vorticity. More specifically, we consider the Benjamin-Feir instability, also called modulational instability. Vorticity can be generated for example by wind blowing above water surface. In our 2-D model, the undisturbed flow is defined by a fluid velocity $u = \Omega y i$ where Ω is the constant vorticity, y is the vertical coordinate, the origin being at the level of liquid at rest and i being an horizontal vector. Bernoulli equation must then be modified and a perturbation method gives a modified nonlinear Schrödinger equation. It is necessary to make the assumption of a constant vorticity to handle the computations within the framework of potential flow. This is a physically acceptable approximation. In the discussion, we introduce the dimensionless vorticity $\bar{\Omega} = \Omega/k_0 c_0$ where k_0 and c_0 are the wavenumber and the phase velocity of the carrier wave, respectively. We show that a positive vorticity increases instability, and that a negative vorticity has a different effect (changing the sign of vorticity is the same as changing the sense of propagation of the waves). First, it decreases the instability of the wave train. This is valid for small negative values $-0.586 < \bar{\Omega} < 0$. Then, the instability increases again $-3.414 < \bar{\Omega} < -0.586$ and the instability decreases again until there is no more instability but even damping for great negatives values of $\bar{\Omega}$.