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Method of "fictitious controls" and solution of inverse problems of the geophysical hydrodynamics using variational data assimilation

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The study and development of computational algorithms for the solution of inverse and data assimilation problems in geophysical hydrodynamics is one of the important contemporary computation and informational problems. Some of such problems and methods to solve them are studied in this work.

1. Let us consider a spherical coordinate system (λ, θ, r), $z = R - r$, $(\lambda \in [0, 2\pi]$, $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}])$, S_R is the sphere of the radius R, Ω is a part of this sphere, the ocean velocity vector is presented in the coordinate form $U = (u, v, w) \equiv (u, w)$, where $u = (u, v)$ is the 'horizontal' velocity vector and w is the 'vertical' velocity. The sea level will be specified by the equation $z = \xi(\lambda, \theta, t)$, where $(\lambda, \theta, R) \in \Omega$, t is the time variable, $t \in [0, \bar{t}]$, $\bar{t} < \infty$, further $f(u) = l + u \sin\theta/(r\cos\theta)$; $n \equiv 1/r$, $m \equiv 1/(r\cos\theta)$, $l = 2\omega\sin\theta$, where ω is the Earth angular velocity.

2. Let us in $D \times (0, \bar{t})$ write the system of the nonlinear hydrothermodynamic equations for the functions u, v, ξ, T, S:

$$\begin{cases} \dfrac{du}{dt} + \begin{bmatrix} 0 & -f \\ f & 0 \end{bmatrix} u - g \operatorname{grad}\xi + A_u u + (A_k)^2 u \\ \qquad = f - \frac{1}{\rho_0}\operatorname{grad} P_a - \frac{g}{\rho_0}\operatorname{grad} \int_0^z \rho_1(T,S)dz' \\ \dfrac{\partial \xi}{\partial t} - m\dfrac{\partial}{\partial x}\Big(\displaystyle\int_0^H \Theta(z) u dz\Big) - m\dfrac{\partial}{\partial y}\Big(\displaystyle\int_0^H \Theta(z)\dfrac{n}{m} v dz\Big) = f_3, \\ \dfrac{dT}{dt} + A_T(q,T) = f_T, \quad \dfrac{dS}{dt} + A_S S = f_S, \end{cases} \tag{1}$$

where: $\Theta(z) \equiv r(z)/R$; $f = (f_1, f_2)$, f_3, f_T, f_S are the functions of the 'internal' sources; $g, \rho_0 = const > 0$; T, S are the functions of water temperature and salinity; P_a is a given function and

$$A_T(q,T) = -(\mu_T \Delta T + \frac{1}{r^2}\frac{\partial}{\partial z} r^2 q) \;\; in \;\; D_0, \quad A_T(q,T) = -(\mu_T \Delta T + \frac{1}{r^2}\frac{\partial}{\partial z}\nu_T r^2 \frac{\partial T}{\partial z}) \;\; in \;\; D \backslash D_0,$$

where D_0 is a subdomain of D.

We consider (1) under some system of boundary conditions (see [2]), among them is the following one:

$$U_n^{(-)} T - \nu_T \frac{\partial T}{\partial z} + \gamma_T (T - T_a) = Q + U_n^{(-)} d_T,$$

Initial conditions are set for the functions u, v, T, S, ξ:

$$u = u^0, v = v^0, T = T^0, S = S^0, \xi = \xi^0 \;\text{ at }\; t = 0, \tag{2}$$

where u^0, v^0, T^0, S^0, ξ^0 are the 'initial data' functions.

One of the important practical case of f in (1) is given by

$$f \equiv g \operatorname{grad} G \tag{3}$$

for a certain scalar function $G = G(\lambda, \theta, t)$.

In this work we consider the initial - boundary value problem for the system (1) for the 'main unknowns' u, v, ξ, T, S and q, Q, ν_T are considered as the 'additional unknouns'. To complete the problem we use the sea temperature observations data on the function T and apply the variational data assimilation procedure.

3. Suppose the only function obtained by processing observation data is the function T_{obs} on $\bar{\Omega} \equiv \Omega \cup \partial\Omega$ and in D at $t \in (0, \bar{t}\,)$. Let the physical meaning of this function be an approximation to the sea temperature function T. A case is admitted when T_{obs} exists only in some subset from $(\Omega \cup D) \times (0, \bar{t}\,)$, whose carrier is denoted m_0.

The statement of the inverse problem is the following: find u, v, ξ, T, S, and q, Q, ν_T such that the system (1) with corresponding initial-boundary conditions and the following relation in $\Omega \times (0, \bar{t}\,)$:

$$m_0(T - T_{obs}) = 0 \tag{4}$$

are satisfied. Then, the relation (4) is reduced to the problem of minimization of a quadratic regularized functional.

To study and solve this problem we introduce the numerical approximations based on splitting methods [4], we propose the special method to solve this problem - "the method of fictitious controls" and apply the optimal control approaches ('variational approaches', 'variational data assimilation procedures', see [1]) and use the iterative algorithms for solving the minimization problem.

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References

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