



Optimal solution error covariances in nonlinear problems of variational data assimilation

Victor Shutyaev (1), Igor Gejadze (2), and Francois-Xavier Le Dimet (3)

(1) Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation (shutyaev@inm.ras.ru),
(2) University of Strathclyde, Glasgow, UK (igor.gejadze@strath.ac.uk), (3) Université de Grenoble, Grenoble, France
(ledimet@imag.fr)

The problem of variational data assimilation for a nonlinear evolution model is formulated as an optimal control problem (see, e.g.[1]) to find the initial condition, boundary conditions or model parameters. The input data contain observation and background errors, hence there is an error in the optimal solution. For mildly nonlinear dynamics, the covariance matrix of the optimal solution error can be approximated by the inverse Hessian of the cost functional of an auxiliary data assimilation problem ([2], [3]). The relationship between the optimal solution error covariance matrix and the Hessian of the auxiliary control problem is discussed for different degrees of validity of the tangent linear hypothesis. For problems with strongly nonlinear dynamics a new statistical method based on computation of a sample of inverse Hessians is suggested. This method relies on the efficient computation of the inverse Hessian by means of iterative methods (Lanczos and quasi-Newton BFGS) with preconditioning. The method allows us to get a sensible approximation of the posterior covariance matrix with a small sample size. Numerical examples are presented for the model governed by Burgers equation with a nonlinear viscous term.

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References:

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