



Eddy parameterisations from data-driven coarse-graining of Lagrangian trajectories

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Eddy diffusivity models for the effects of unresolved fluid motion are in common usage for tracer advection in ocean models. These models can be derived by assuming a scale separation in the velocity field and representing the small-scale velocity components by some sort of stochastic model. Following the standard theory of homogenisation, it can be shown that (under suitable assumptions about the small-scale components) Lagrangian trajectories may be approximated by a coarse-grained drift-diffusion stochastic differential equation, and the (possibly spatially-varying) coarse-grained drift and diffusion parameters can be used as the velocity and eddy diffusivity for the coarse-grained tracer transport. Extending this approach to higher-order models designed to capture sub- and super-diffusive behaviour (such as those proposed in Griffa (1996)) gives rise to multi-component subgrid models for tracer transport. In this work we take a data-driven approach, and are concerned with how to extract these coefficients from observational data or high-resolution ocean model output. These coefficients can then be used to benchmark eddy parameterisations (such as the Gent-McWilliams parameterisation), to diagnose mixing, or even inserted directly into ocean models. For simple idealised flows for which the eddy diffusivity can be calculated analytically using homogenisation theory, we have shown that a maximum likelihood estimator for computing the eddy diffusivity from Lagrangian trajectories converges in a statistical sense to correct value, provided that the trajectories are long enough. However, the data must be subsampled in order to filter out the multiscale error that arises because the coarse-grained eddy diffusion model is only consistent with the data at the coarse-grained scale (Cotter and Pavliotis, *Commun. Math. Sci.* 2009). We have also shown that naïve attempts to reduce the error in the estimators by making use of the discarded data fail, and possibly even introduce bias. We propose a more sophisticated way to make use of all of the data based on a shuffling algorithm that turns the multiscale error into decorrelated noise without altering the coarse-grained coefficients. We also show how this approach can be extended to spatially-varying coarse-grained coefficients, and illustrate with idealised flows and results from a barotropic gyre.