



Self-similar evolution of wind wave statistical momenta

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The ultimate aim of studies of random wind waves is to predict probability density function of wave characteristics, primarily wave height, at any given place and time. We study long-term evolution of typical random wind waves, which are characterized by broad-banded spectra and quasi-Gaussian statistics. We find the departure of wave statistics from Gaussianity from first principles, using higher-order statistical momenta (skewness and kurtosis) as a measure of this departure. Non-zero values of kurtosis mean an increased or decreased probability of extreme waves (compared to that in a Gaussian sea), which is important for assessing the risk of freak waves and other applications.

For water waves, there are two different contributions to kurtosis. The first one is due to bound harmonics, while the second one is linked to nonlinear wave-wave interactions. The latter contribution requires information on the phases of interacting waves and, therefore, can be found only with direct numerical simulation. We perform such a simulation of wind-generated random wave fields and swell, using a specially designed algorithm based on the Zakharov integrodifferential equation for water waves. In a generic situation, the contribution to kurtosis due to wave interactions is shown to be small compared to the bound harmonics contribution. This observation enables us to determine higher momenta by calculating the bound harmonics parts directly from spectra using asymptotic expressions. Thus, the departure of evolving wave fields from Gaussianity is explicitly contained in the wave spectra. This enables us to broaden significantly the capability of the existing systems for wave forecasting: in addition to simulation of spectra, it becomes possible to simulate also higher momenta. We found that the contributions due to bound harmonics to both skewness and kurtosis are significant for oceanic waves, and non-zero kurtosis (typically in the range 0.1–0.3) implies a tangible increase of freak wave probability.

From the analysis of the wave kinetic equation, it is well known that wave spectra of a developing wind wave field and swell often evolve in a self-similar manner. For such self-similar regimes we derive asymptotic formulas for skewness and kurtosis of a random wave field generated by constant wind, and for swell. In particular, for the Zakharov–Zaslavsky regime of growing wind sea, corresponding to constant momentum flux to waves, we obtain the asymptotic decrease of kurtosis and skewness with time, proportional to $t^{-4/11}$ and $t^{-2/11}$ respectively. These asymptotic properties are verified by direct numerical simulation.