



On Earth's Gravity Field recovery from Satellite Orbit Perturbations

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Earth satellites orbits are to a great extent controlled by the Earth's gravity field. It is the biggest force contributor in the equations of motion. Any irregularities in the Earth's gravity field are directly mapped in the orbital shape. Therefore one might say that irregularities in Satellite orbits are strongly correlated with irregularities in the Gravity field. It is then very much clear that if one is able to find appropriate mathematical and physical relation between these orbit irregularities and Gravity field parameters, the gravity field might be determined indirectly from these orbital perturbations on a global scale. The spatial data distribution, data quality and parameters chosen control the Gravity field extent we recover in terms of wavelength, for example if as parameters we choose the solid spherical harmonics. Once having the gravity potential in terms of its parameters, its derivatives might be computed, for example gravity anomalies or geoid undulations.

Main focus of attention of this paper is detailed mathematical formulation of the recovery of the Earth's gravity field parametrized by normalized spherical harmonics coefficients from satellite orbit data. Basic motivation is to give useful, complete and clear explanation in terms of mathematical formulas usable for further programming or practical implementation. As a baseline of this paper, the "classical" full matrix numerical integration of the orbit and variational equations will be used and explained. All formulas presented deal with fully normalized spherical harmonics, and the gravity potential is represented as a function of cartesian coordinates. In this sense the overall computation is much more elegant and simpler. As a starting baseline in this approach the Cunningham (1970) and Montenbruck (2005) will be followed and their derivations are modified here to be able to deal with normalized spherical harmonics.

In addition to that a test scenario will be presented, where spherical harmonics up to and including degree and order 140 are recovered from the orbit. Furthermore, some consideration on the numerical stability of the normal equations system is given together with focus on proper length of arcs, numerical integration accuracy, data gaps, parameters correlation as well as proper treatment of the observations noise. At the end, a test scenario will be presented with C++ double double precision implementation showing dramatic improvement in the orbit integration (nanometer over 24h integration accuracy) and the spherical harmonics determination (6 orders of magnitude improvement in the gravity field determination) as well as in the normal equations stability as compared with the double precision computations. This do not only improves the overall accuracy but also opens the door for this approach to be used in the current and future interferometric satellite missions without any degradation due to the numerics. All tests presented are computed by our own software ARCSST for orbital integration and analysis.