



Implementation of high-order particle-tracking schemes in a water column model

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Stochastic differential equations (SDEs) offer an attractively simple solution to complex transport-controlled problems, and have a wide range of physical, chemical, and biological applications, which are dominated by stochastic processes, such as diffusion. As for deterministic ordinary differential equations (ODEs), various numerical scheme exist for solving SDEs. In this work, 10 particle-tracking schemes are presented and tested for accuracy and efficiency (time vs. accuracy). To test the schemes, the particle tracking algorithms are implemented into a community wide used 1-D water column model (General Ocean Turbulence Model, GOTM). The various tracking schemes are validated against known analytical solutions.

Modelling individual particles allows a straightforward physical interpretation of the involved processes. Further, this approach is strictly mass conserving and does not suffer from the numerical diffusion that plagues grid-based methods. Moreover, the Lagrangian framework allows assigning properties to the individual particles, that can vary spatially and temporally. The movement of the particles is described by a stochastic differential equation, which is consistent with the advection-diffusion equation. Here, the concentration profile is represented by a set of independent moving particles, which are advected according to the velocity field, while the diffusive displacements of the particles are sampled from a random distribution, which is related to the eddy diffusivity field.

The work will show that especially the 2. order schemes are accurate and highly efficient. At the same level of accuracy, the second-order scheme can be significantly faster than the first-order counterpart. This gain in efficiency can be spent on a higher resolution for more accurate solutions at a lower cost.

Special attention is given to the noise increments and hence the random variables. The analysis showed that with a careful choice of the random number generator used, a speed improvement by a factor of three, compared to default generators, can be achieved. The analysis further indicated that random numbers, drawn from two-point of three-point distributions, could provide additional speed gains. By doing so, care has to be taken, that the use of these discrete random distributions is limited in the case of diffusion from point sources or in the short term dynamics.

The motivation of this work is to find efficient numerical solvers that fit the needs of the ocean modelling community. Having these implementations, they can be adapted to problems in biology, turbulence, or sediment modelling. Moreover, the numerical scheme can be extended to include effects of higher order moments in the fluctuation statistic, temporal and spatial correlations, inertia effects, or non-Gaussian velocity statistic. Furthermore, a 1-D particle-tracking model can act as the inner core of a full 3-D particle tracking code. The intention is not to replace concentration-based models, but rather gain additional insights in the processes involved.