



Stationary runup patterns on a sloping beach

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The key topic of this presentation is the effect of obliquity on the runup of waves on an inclined plane. Well-controlled runup solutions, obtained under idealized circumstances, are important both for model validation and for gaining insight into the dynamics of the runup phenomenon as such.

When a wave of permanent form is obliquely incident on an inclined plane the wave pattern becomes stationary in a frame of reference which moves along the shore. This enables a simplified mathematical description, which has been exploited in a few published articles on wave reflection and runup and will be employed also in the present study.

Using linear shallow water theory Carrier & Noiseux (1983) investigated the reflection of Gaussian shaped pulses, representing tsunamis, from straight coastlines and continental shelves. Assuming nearly normal incidence Ryrie (1983) developed weakly three-dimensional nonlinear shallow water equations for propagation and runup of bores. Brocchini & Peregrine (1996) combined the approximation of weak obliquity with the hodograph transformation for the motion in the direction normal to a plane beach and investigated the along-shore flow for periodic incident waves. Later (1998) the study was extended to solitary waves. In both these works the along-shore flow was decoupled from the motion in the normal direction, which was governed by the standard equation set for plane waves of normal incidence.

Since there is no coupling back from the along-shore to the normal flow such a procedure reproduces the runup of normal incidence.

No study of this kind with strong obliqueness, and hence the variation of the runup heights with the angle of incidence, is available in the literature.

In the present study the stationarity of the wave patterns is exploited to develop equations including strong obliqueness, nonlinearity and weak dispersion. These equations are used to compute runup heights.

First, a nonlinear and weakly dispersive set of Boussinesq equations for the downstream evolution of stationary wave patterns is derived. In the hydrostatic approximation streamline based Lagrangian versions of the evolution equations are developed for automatic tracing of the shoreline. Both equation sets are, in their present form, developed for non-breaking waves only. Finite difference models for both equation sets are designed. These methods are then coupled dynamically to obtain a single nonlinear model with dispersive wave propagation in finite depth and an accurate runup representation. A major gain, in comparison to solving corresponding equations in two horizontal dimensions and time, is the reduction of the problem to involve only one spatial and one time like independent coordinate.

This again allows for huge computational domains, combined with very high resolution, and makes complicated seaward input/radiation conditions superfluous.

The models are tested by runup of waves at normal incidence and comparison with a more general model for the refraction of a solitary wave on a slope. Finally, a set of runup computations for oblique solitary waves is performed and compared with estimates of oblique runup heights obtained from a combination of an analytic solution for normal incidence and optics. We find that the runup heights decrease in proportion to the square of the angle of incidence for angles up to 45 degrees, for which the height is reduced by around 12% relative to that of normal incidence.