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Nonlinear water wave interactions

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Our concern in this abstract is the problem of water waves in a free surface, the form of solitary wave solutions and wavetrains, and their behavior under interactions such as collisions. The relevance to ocean dynamics is well known, and in particular the nonlinear effects of wave interactions are considered to be fundamental to large amplitude wave dynamics, to wave breaking, and to the character of ocean wave spectra. In this talk we will focus on the above two particularly clean cases of nonlinear wave interactions, which are amenable to study with numerical simulations and wave tank experiments, and in particular with mathematical analysis.

Solitary waves and periodic wavetrains for the Euler equations have been discussed in the scientific literature since the time of Stokes. In a long wave perturbation regime and in case of two dimensional motions (that is, of solutions which are essentially constant transverse to the direction of motion) these are well described by single soliton solutions and periodic elliptic function solutions of the Korteweg deVries equation (KdV). The two cases under discussion concern the analog of these solutions for the case of dynamics for the full Euler equations, and for both two and three dimensional fluid motions.

It is a famous result that multiple soliton solutions of the KdV exhibit elastic collisions. The first question that we address is as to what extent interactions between Stokes solitary wave solutions of the Euler equations deviate from being elastic. In this talk I will present numerical, experimental and analytical results on this question, concerning both co-propagating and counter-propagating cases of large amplitude solitary waves. In all cases we find evidence of inelastic interactions, including information on the nonlinear run-up of colliding waves, their phase offset, and the appearance of a dispersive trailing wave which is a residual product after the encounter. However it is remarkable that collisions of even large solitary waves are very close to being elastic, and that the residual is very small [1][2].

The second question that we address concerns periodic traveling wave patterns in the free surface. The character of wave patterns is quite different in the case of a fluid region of large depth, as opposed to the case of shallow water. We will highlight these differences, along with a further description of the nonlinear run-up that arises from the interaction of two-dimensional wavetrains at an oblique angle [3]. Among other things, elements of this process are reminiscent of the phenomenon of focusing which is related to the generation of rogue waves.

References

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