



Robust ensemble filtering in handling uncertainties in data assimilation

Xiaodong Luo and Ibrahim Hoteit

King Abdullah University of Science and Technology, Saudi Arabia (xiaodong.luo@kaust.edu.sa)

Sequential data assimilation methods, including the ensemble Kalman filter (EnKF) and the particle filter (PF), traditionally apply the Bayes' rule to update the background statistics. In order to achieve the prescribed optimality under the Bayes' rule, the complete knowledge of the statistical properties (for example, the pdfs) of both the mathematical model and the corresponding observations is needed, and one also has to assume the assimilation algorithm is able to attain the exact optimal estimates even in the presence of nonlinearity. In reality, however, often neither of the above two assumptions is satisfied. Instead, uncertainties arise from several sources, including errors in specifying initial conditions, the model error, the observation error, and so on. As a result, it is not unusual that a Bayesian filter fails to achieve a reasonable performance. For instance, if implemented straightforwardly, the EnKF may suffer from filter divergence, and it is customary to introduce covariance inflation to overcome this problem.

In this contribution, we propose a robust ensemble filtering scheme based on the H_∞ filtering theory. The optimal H_∞ filter uses the minimax rule to update the background estimates. By design, it is more robust than the Kalman filter in the sense that the estimation error of its analysis has a finite growth rate with respect to the uncertainties in assimilation, except for a special case that corresponds to the Kalman filter (KF). The original form of the H_∞ filter contains global constraints in time, which may be inconvenient for sequential data assimilation problems. Therefore, we introduce a variant that solves some time-local constraints instead, and hence we call it the time-local H_∞ filter (TLHF). By analogy to the EnKF, we also propose the concept of ensemble time-local H_∞ filter (EnTLHF). We outline the general form of the EnTLHF, and discuss some of its special cases. In particular, we show that an EnKF method with a certain covariance inflation technique is essentially an EnTLHF. In this sense, the EnTLHF provides a general framework for conducting covariance inflation in the EnKF-based methods. We use two numerical examples to assess the relative robustness of the TLHF/EnTLHF in comparison with the corresponding KF/EnKF method without covariance inflation.