



Probability density Functions for advective-diffusive transport in heterogeneous porous media

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Our work considers the process of non-reactive solute transport in randomly heterogeneous porous media under advective-diffusive conditions. We develop equations satisfied by the probability density function (PDF) of the concentration of the solute. Our work is motivated by the fact that proper description of flow and transport in heterogeneous porous media and adequate understanding and reliable estimation of the associated uncertainty require stochastic approach. Key points of a stochastic analysis include providing information about moments and, ultimately, probability density distributions. It is clear that merely describing the behavior of the first few statistical moments is not enough for predicting rare events that are described by the tails of statistical distributions. One should then examine the space-time evolution of the probability density function (PDF) of solutes concentrations, $c(x,t)$, or the set of highest moments of c . The advective-diffusive / advective-dispersive problem has only been marginally studied. As diffusive/dispersive processes play a significant role in contaminant mixing and spreading in aquifers, we mainly focus on this aspect.

We introduce the random functional $p(x, t; c)$, representing the density distribution function (DDF) of the local random concentration $c(x, t)$ in the one-dimensional phase space of its possible values C , where x is multi-dimensional vector of spatial coordinates, and t is time. By using the stochastic transport equation in the (x, t) space, one can write the stochastic equation for the functional $p(x, t; c)$. Averaging this equation in the $(x, t; C)$ space leads to an equation for the probability density function (PDF) for $c(x, t)$, $P(x, t; c) = \langle p(x, t; c) \rangle$ (where angular brackets indicate expected value operator), and the corresponding power moments. The diffusive term in the equation satisfied by p gives rise to two similar diffusive terms, respectively in the x - and C -space.

For the case where diffusion is described by a constant coefficient, D , the diffusive term in the x -space has a traditional Fickian form. We have then developed an expression for the diffusive term acting in the concentration space and analyzed it in the context of a simple, one-dimensional set-up.