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Combined approach to a simulation of the interaction of nonlinear spatial waves in seas with gently sloping bottom

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This paper deals with the development of a new method for the evolution description of moderately long three-dimensional nonlinear perturbations of the water surface. It is supposed that a stationary flow is absent, the disturbance amplitudes are small but finite, characteristic horizontal lengths of waves and of the bottom topography are larger than the water depth h. The initial system of the hydrodynamic equations for the shallow water above the gently sloping bottom was reduced to one basic nonlinear evolution differential equation for spatial perturbations of the surface η: $\partial^2\eta/\partial t^2 - gh\nabla^2\eta - (g/2)\nabla^2\eta^2 - h\nabla^2\left(\mathrm{u}^2\right) - g\left(\nabla\cdot h\right)\left(\nabla\cdot\eta\right) - \left(h^2/3\right)\nabla^2\left(\partial^2\eta/\partial t^2\right) = 0$, and two linear auxiliary differential equations for a determination of the horizontal velocity vector **u** averaged over the layer depth, which is contained in the main equation only in one term of the second order of smallness: **u**$= \nabla\varphi$ and $\nabla^2\varphi = -\left(\partial\eta/\partial t\right)/h$. Here t is the time, g is the free fall acceleration, the operator $\nabla = (\partial/\partial x, \partial/\partial y)$, $(x,\ y)$ is the horizontal plane, and φ is the potential of the water velocity. The suggested model (the first version was published in the article [1]) is suitable for finite-amplitude waves running simultaneously in different directions.

For the solution of the main nonlinear equation a modification of the implicit three-layer finite difference scheme is constructed. The scheme is similar to the one described in the article [2]. Calculations according to this scheme have been carried out with Seidel's iteration algorithm in the horizontal plane. Poisson's equation for determination of the velocity vector was solved by the method of the fast Fourier transformation in both horizontal coordinates at each step of time.

Some solutions of the model system were found numerically with the help of this method. For example, it was considered the case of the evolution of the initial peakless cross-type disturbance (the superposition of two plane perturbations propagated in the direction of growth of the x-axis, as well as of the y-axis): $\eta = \eta_x$ if $\eta_x > \eta_y$ and $\eta = \eta_y$ if $\eta_y > \eta_x$, where $\eta_i = \eta_{ia}/\cosh^2[(x - U_i t)/(4L_i)]$, $I = x, y$, $U_i = \sqrt{gh\left(1 + \eta_{ia}/3h\right)/\left(1 - 2\eta_{ia}/3h\right)}$, $L_i = 2h\sqrt{\left(1/3 + h/\eta_{ia}\right)/3}$ (U_i and L_i are the velocity and the length of the plane soliton solutions). In this case an appearance of a peak at the cross of two wave fronts with time is observed. In particular, η_{max} = 0.312 h if η_{xa} = 0.15 h and η_{ya} = 0.1 h for $t = 100\sqrt{h/g}$, that is η_{max} = 1.25 η_s, where $\eta_s = \eta_{xa}$ + η_{ya}. (η_{max} = 0.6 η_s for t = 0). When we calculated this problem with the help of the two-dimensional Boussinesq equation, we have obtained that η_{max} = 1.15 η_s. And when we solved this problem with the help of Kadomtsev–Petviashvili equation (the cross-type disturbance running approximately along the bisector of the angle $x0y$), we have obtained that η_{max} = 1.07 η_s. From the mathematical standpoint these equations allow studying interactions of disturbances propagated on any angles. But these equations were derived with the assumption that nonlinear perturbations travel chiefly in one direction. Besides KP model proposes that in such wave processes a characteristic horizontal scale along the longitudinal coordinate is less than along the transversal one. Therefore in our problem this model unable to describe correctly not only a nonlinearity but also a dispersion.

REFERENCES

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