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An R&D Program to Solve the Flux Paradox: Solving Calderón's Problem combined with Kalman Filtering

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Knowledge of the time-dependent locations and intensities of infiltrating and exfiltrating groundwater fluxes is primordial for an adequate management, protection, and sustainable use of groundwater. Since the 1960s groundwater models have become an indispensable tool for groundwater management. Such models are based on physical laws, like continuity and Darcy's law, as well as on a 3-dimensional hydrogeological conceptual picture of the hydraulic conductivities of the subsurface. It is almost an article of faith to claim that such models can handle only one boundary condition on the phreatic surface: either the head (the Dirichlet problem), or the flux (Neuman problem). The phreatic heads can be measured in observation wells with relatively good accuracy compared to the fluxes, which can only be "measured" indirectly from measured precipitation, evapotranspiration and specific yield times rate of water table rise. One would, therefore, expect that the time-dependent head is the preferred phreatic boundary condition. The groundwater fluxes then follow from the model calculations. This way we have found one of the most important quantities that we need for adequate management. Unfortunately, when doing so, the calculated fluxes may appear so unrealistic that hydrogeologists are forced to use fluxes as phreatic boundary conditions. The fluxes that we are looking for have to be specified... Is there a way to overcome this "flux paradox"?

Hydrogeologists are generally well acquainted with the "Dirichlet problem" and the "Neuman problem." However, the "Calderón problem" is virtually unknown in the hydrogeological community. Yet, the Calderón problem may become one of the keys to solve the flux paradox. In the Calderón problem both the head and the flux are specified on (parts of) the boundary. When doing so, it is possible—under some conditions—to determine the groundwater heads and flow rates, as well as the hydraulic conductivities in the 3-dimensional subsurface or aquifer-aquitard system.

The Kalman Filter may be the second key to solve the flux paradox. During a relatively dry period time the infiltration-exfiltration fluxes are mainly determined by specific yield times rate of water table decline, which can be determined with reasonable accuracy. Let us assume that we can do so during a period of a quarter of a year (91 days). For each day we solve the Calderón problem. This way we obtain a time series of "observed" hydraulic conductivities. Because of the measurement noise, the thus obtained 91 spatial conductivity distributions may differ from day to day, from which the observation noise can be determined. From the relatively noisy observed conductivities (observed by solving the Calderón problem 91 times) the Kalman Filter estimates the "true" conductivities with an estimation noise that is appreciably smaller than the observation noise. After 91 days the Calderón-Kalman Filter procedure is terminated and the thus obtained hydraulic conductivities are considered as the best possible estimate. Using these conductivity estimates, the fluxes for other time periods—for which such accurate flux measurements are not available—can be determined by solving a conventional Dirichlet problem based on phreatic heads measured in observation wells.

The above-presented idea is still part of an R&D program. The flux paradox has not yet been solved. However, encouraging results have already been obtained, as will be shown during the presentation.