



## Macro-permeability $K$ of highly fissured porous media : inertial and percolation effects

Rachid Ababou (1), David Bailly (2), and Michel Quintard (3)

(1) Institut de Mécanique des Fluides de Toulouse, Toulouse, France (ababou@imft.fr), (2) Laboratoire Ecolab & Institut de Mécanique des Fluides de Toulouse, Toulouse, France, (3) Institut de Mécanique des Fluides de Toulouse, Toulouse, France

[preprint,12pt]elsarticle amssymb amsmath epsfig amssymb,amsbsy,amsmath,amsfonts,amssymb,amscd latexsym,euscript,exscale,epsfig epsfig [english,français]babel times [latin1]inputenc [T1]fontenc

### Macro-permeability $K$ of highly fissured porous media : inertial and percolation effects

D. Bailly<sup>1,2</sup>, R. Ababou<sup>2</sup>, M. Quintard<sup>2</sup>

<sup>1</sup>Laboratoire Ecolab, Ecole Nationale Supérieure Agronomique de Toulouse, Avenue de l'Agrobiopôle, 31326 Castanet-Tolosan, France.  
 E-mail: bailly@ensat.fr; davidbailly31@hotmail.com

<sup>2</sup>Institut de Mécanique des Fluides de Toulouse, Allée du professeur Camille Soula, 31400 Toulouse, France.  
 E-mail: ababou@imft.fr; quintard@imft.fr

#### ABSTRACT

To study highly fissured 3D media such as karsts – and other kinds of heterogeneous and discontinuous media – a numerical code *MatFrac3D* has been developed (Bailly 2009; Bailly et al. 2009). The *MatFrac3D* code can generate hybrid matrix/fissure media, with a spatially distributed matrix plus discrete fissures represented by 'objects' of various shapes and sizes (e.g. thin planar discs, cylindrical conduits, spheroidal cavities). The synthetic media are then interfaced with a flow code *BigFlow3D-Python*, dedicated to flow simulation in highly heterogeneous or stochastic porous media (Ababou et al. 1993; Ababou et al. 1996; Albitar et al. 2005), in order to analyze their macroscale hydraulic behavior (focusing here on saturated flow).

We present numerical flow experiments conducted under "permeametric", and "immersion" boundary conditions. For the local scale equations, we use the linear / quadratic head loss law, that combines additively the linear law of Darcy 1856 and the quadratic law of Ward 1964 and Forchheimer 1930. We name it 'Darcy / Ward-Forchheimer', or 'DWF'. Specifically, the following 'DWF' head loss law was implemented in the *BigFlow 3D* code, for the case of diagonally anisotropic media (Trégarot 2000; Ababou et al. 2002):

$$q_i^{DWF} = -\tilde{K}_{ii}^{DWF}(h, \vec{\nabla}H, \vec{x}) \frac{\partial H}{\partial x_i} \quad (1)$$

$$\tilde{K}_{ii}^{DWF}(h, \vec{\nabla}H, \vec{x}) = \frac{2K_{ii}^D(h, \vec{x})}{1 + \left\{ 1 + 4\gamma K_{ii}^D(h, \vec{x}) \left( \sum_i \sum_i \frac{\partial H}{\partial x_i} K_{ii}^D(h, \vec{x}) \frac{\partial H}{\partial x_i} \right)^{1/2} \right\}^{1/2}} \cdot$$

where  $q_i^{DWF}$  is the DWF velocity,  $\gamma = C/(g\nu)^{1/2}$  with  $C = 0.55$  (dimensionless Ergun factor),  $K_{ii}^D(\vec{x})$  are the components of the (diagonal) Darcian conductivity tensor [ $m \cdot s^{-1}$ ], and  $i = (1, 2, 3)$ . Note: repeated indices should not be summed in eq.?? unless explicitly shown with the  $\sum$  symbol.

In the first part of this work, we focus on upscaling Darcy's law (the linear part of the DWF law). The numerically upscaled permeability, named here macro- $K^D$ , is compared with two analytical approximations: *self-consistent* (Dagan 1981) and *power averaging* (Ababou 1996: Appendix B). In addition, we analyse the sensitivity  $\partial_\kappa \log_{10} K^D$  of the macro- $K^D$  with respect to the fissure(F) / matrix(M) permeability ratio  $\kappa = \log_{10} K_F / K_M$ . The sensitivity  $\partial_\phi \log_{10} K^D$  of the macro- $K^D$  is also analysed with respect to the volume fraction of fissures,  $\phi$ . These analyses lead to a characterisation of critical transitions and quasi-percolation thresholds for the macro- $K^D$  vs.  $\phi$ . Finally, we also study the influence of numerical parameters on these results (mid-nodal conductivity weighting scheme; mesh resolution of embedded objects or fissures).

In the second part, we focus on fast flow regimes / inertial effects. Numerical experiments are conducted using DWF as the local head loss law (eq.??), for a broad spectrum of Reynolds numbers  $Re_{DWF}$  up to 1000. Optimal calibration procedures are developed in order to identify the inertial head loss law at the macroscale. A generalized DWF power law emerges as follows (tested here under permeametric conditions):

$$-\vec{\nabla}H = \left\{ B_1 + A_1 |\vec{q}_{DWF}|^\beta \right\} \vec{q}_{DWF}, \quad (2)$$

where the exponent  $\beta$  and the coefficients  $B_1$  and  $A_1$  are three optimised parameters (best fit). In eq.??, the term between brackets  $\{ \}$  can be interpreted as the equivalent DWF *macro-resistivity* (inverse macro- $K^{DWF}$ ), which increases with Reynolds number due to inertial head losses. For example, in the case of a perfectly stratified medium with flow parallel to strata, the power law fit was quite good, and the resulting exponent was roughly  $\beta \approx 0.90 - 0.92$  for Reynolds numbers ( $Re_{DWF}$ ) in the range 100–1000. In comparison, for  $Re_{DWF} \leq 10$ , the fit yielded  $\beta \approx 1.00$ , i.e., same as in the assumed local scale DWF law (as expected).

In summary, the macroscale permeability of highly fissured porous media was studied, including: (i) an analysis of quasi-percolation effects in the Darcian regime; (ii) a study of permeability anisotropy and of its tensorial nature (in the Darcian regime); and (iii) a study of inertial fast flow effects in terms of a macroscale, Reynolds dependent, equivalent resistivity.