



Geomagnetic data assimilation with an implicit particle filter into a one-dimensional sparsely observed MHD system

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It is known that secular variations of the geomagnetic field are linked to the velocity field in the core. A good numerical model thus has to account for this coupling. While the magnetic field has been measured for the past few hundred years (and with improving accuracy), no measurements are available for the velocity field. Recent research efforts aim at using magnetic data to obtain better model predictions of the core velocity through data assimilation. The data assimilation problems are presented in terms of a stochastic differential equation (SDE) and, mathematically, the task is to estimate the state $x(t)$ of the SDE, with additional information provided by noisy observations b^n , $n = 1, 2, \dots$. The state estimate is typically a statistic (often the mean) of the conditional probability density function (pdf) $p_{n+1} = p(x(t^{n+1})|b^0, \dots, b^n)$. Thus far, a Kalman filter and variational methods have been applied to geomagnetic data assimilation. However, there are problems with both approaches (the Kalman filter relies on a Gaussian assumption and the computations required for variational methods can be prohibitively expensive).

Particle filters represent another class of sequential data assimilation methods, which are well suited for nonlinear, non-Gaussian problems, but have not been applied in geomagnetics. In short, the procedure in traditional particle filters is as follows: at each time t_n , a prior density is constructed by following replicas of the model (called particles). The prior is updated by sampling weights determined by the observations b^{n+1} , to yield a posterior density that approximates p_{n+1} . Because the observations are not used when constructing the prior, particles paths are likely to stray into regions of low probability and the number of particles required can grow catastrophically, especially if the dimension of the SDE is large (as is the case in geomagnetic data assimilation). The implicit particle filter is a new particle filter that reverses the standard procedure by first assigning a probability to each particle and then finding a sample that assumes it. This reversed procedure focusses the particles towards the observations and generates a thin particle beam within the high probability domain. Numerical experiments verify that the implicit filter performs well when solving large dimensional data assimilation problems.

We explore the potential of the implicit filter in geomagnetic data assimilation by applying it to a sparsely observed, magneto-hydrodynamic (MHD) system as proposed by Fournier et al. (A. Fournier, C. Eymin, T. Alboussiere, *A case for variational geomagnetic data assimilation: insights from a one-dimensional, nonlinear, and sparsely observed MHD system*, Nonlin. Processes Geophys., 14, pp. 163-180, 2007). The model is simple enough to test our filter in detail, but retains some important features from the induction and Navier-Stokes equations. We deploy a network of 'stations' along our numerical grid to observe the magnetic field at various locations, but keep the velocity unobserved. We vary the number of observation locations to test how sparsely we can observe the magnetic field, while still improving the model prediction for the velocity field in the core. We compare our results to those obtained by Fournier et al. with a variational approach and to those obtained by a traditional particle filter.