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Three-Dimensional Simulation of Time-Domain Electromagnetic Fields using Krylov Subspace Methods, Finite Elements, and an Exact Boundary Condition

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We present a numerical method for the simulation of time-domain electromagnetic fields in arbitrary three-dimensional conductivity distributions.

The spatial discretization of the underlying boundary value problem is done using the finite element technique employing Nédélec elements on an unstructured tetrahedral grid. Our method allows us to restrict the computational domain to the conducting lower halfspace. The effect of the insulating upper halfspace can be incorporated in terms of an exact non-local boundary condition at the Air-Earth interface.

It is well known that standard numerical time integration methods such as Euler's explicit time stepping method require extremely small time steps to preserve stability of the solution in time. However, for the simulation of a typical time-domain electromagnetic sounding, only a small number of data points is of interest, which are spread over several decades in time. Due to the huge numerical effort required to advance the simulated fields to late time stages, time stepping methods are of little attraction when computational efficiency is a crucial requirement. The latter is of particular interest for inversion methods which aim at the reconstruction of the spatial distribution of electrical conductivity within the lower halfspace.

We demonstrate a substantial improvement in the numerical efficiency of the time integration by using Krylov subspace based methods.

The spatial discretization of the continuous boundary value problem yields a large, sparse and symmetric matrix A which, together with the time derivative operator, acts on the electric field discretized in space. This forms a system of ordinary differential equations which represents the semi-discretized form of the parabolic partial differential equation.

The solution to such an initial value problem can be explicitly expressed by a matrix function f(A) multiplied with a vector of initial values b. In our case, f(A) is the matrix exponential exp(tA) with t being an arbitrary time after current shut-off and b representing the discrete electric field at a short time after current shut-off. The direct evaluation of exp(tA) is, however, too expensive due to the size of A.

There are several techniques available to evaluate f(A)b sufficiently accurate with high numerical efficiency. One of the classical methods is the Spectral Lanczos Decomposition Method (SLDM) which employs the Lanczos method to construct an orthonormal subspace basis of a Krylov space initialized from A and b. After projection of A onto that small Krylov space, the approximate evaluation of exp(tA) becomes feasible. The drawback of SLDM is that the complete subspace basis is required for a successful approximation to f(A)b. A viable alternative to SLDM is the restarted SLDM which is considerably faster and more efficient in terms of memory requirements. Further improvements have been investigated by using rational Krylov methods which describe the spectrum of eigenvalues of A.

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