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The evaluation of different formulations of the MT boundary value problem for 3D finite element simulation

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Maxwell's equations give rise to different formulations of the three-dimensional (3D) magnetotelluric (MT) boundary value problem. Assuming a harmonic time dependency $e^{i\omega t}$ of the electromagnetic fields the equation of induction can be derived in terms of
(i) the magnetic field $\mathbf{H}$

$$\nabla \times (\sigma + i\omega\varepsilon)^{-1}\nabla \times \mathbf{H} + i\omega\mu\mathbf{H} = 0, \tag{1}$$

(ii) the electric field $\mathbf{E}$

$$\nabla \times \mu^{-1}\nabla \times \mathbf{E} + (i\omega\sigma - \omega^2\varepsilon)\mathbf{E} = 0, \tag{2}$$

(iii) the magnetic vector potential $\mathbf{A}$ and the electric scalar potential ϕ

$$\nabla \times \mu^{-1}\nabla \times \mathbf{A} + (i\omega\sigma - \omega^2\varepsilon)\mathbf{A} + (\sigma + i\omega\varepsilon)\nabla\phi = 0, \tag{3}$$

(iv) the magnetic vector potential $\tilde{\mathbf{A}}$ only

$$\nabla \times \mu^{-1}\nabla \times \tilde{\mathbf{A}} + (i\omega\sigma - \omega^2\varepsilon)\tilde{\mathbf{A}} = 0, \tag{4}$$

(v) the anomalous magnetic vector potential $\mathbf{A}_a$

$$\nabla \times \mu^{-1}(\nabla \times \mathbf{A}_a - \mu_a\mathbf{H}_n) + (i\omega\sigma - \omega^2\varepsilon)\mathbf{A}_a = (\sigma_a + i\omega\varepsilon_a)\mathbf{E}_n \tag{5}$$

with the magnetic permeability $\mu = \mu_0\mu_r$ (μ_0 - magnetic field constant, μ_r - relative magnetic permeability), the dielectric permittivity $\varepsilon = \varepsilon_0\varepsilon_r$ (ε_0 - electric field constant, ε_r - relative dielectric permittivity), the electrical conductivity σ, the angular frequency ω and the imaginary unit i. $\mathbf{E}_n$ and $\mathbf{H}_n$ denote the normal electric and magnetic fields, respectively, that arise from the one-dimensional layered halfspace solution with σ_n, ε_n and μ_n. In this case, 3D structures are represented by anomalous electromagnetic parameters σ_a, ε_a and μ_a so that $\sigma = \sigma_n + \sigma_a$, $\varepsilon = \varepsilon_n + \varepsilon_a$ and $\mu = \mu_n + \mu_a$. Including Dirichlet and Neumann boundary conditions, the vector fields can be calculated in a bounded domain Ω. In the case (iii) approximating the vector and the scalar potential the equation of continuity

$$-\nabla \cdot \left((i\omega\sigma - \omega^2\varepsilon)\mathbf{A} + (\sigma + i\omega\varepsilon)\nabla\phi\right) = 0 \tag{6}$$

is needed additionally to determine all unknowns. From the spatial derivatives of the vector fields we obtain MT data such as the impedance tensor, apparent resistivity, and phase.

To numerically solve the boundary value problems we apply the finite element (FE) method on unstructured tetrahedral grids. Vector finite elements ensure continuity of the tangential field components and allow for discontinuous normal field components at parameter contrasts. Therefore, they are well suited to approximate 3D electromagnetic quantities. Their degrees of freedom (DOF) are associated with the edges, faces and the volume of the tetrahedra. A direct solver is utilised to solve the system of equations which results from the FE discretisation. Differentiation of the FE shape functions representing the approximate behaviour of the vector fields in space enables the computation of the MT data.

We present a comparison of the different formulations of the MT boundary value problem concerning their accuracy and numerical efficiency. Since Maxwell's equations are not perfectly symmetric the boundary value problem for the magnetic field shows other characteristics than those for the electric field and the potential approaches.

Beside the comparison with analytical solutions, convergence studies provide estimates of accuracy especially for more complicated models for which an analytical solution does not exist. In order to evaluate not only the discretisation error but also the interpolation error that depends on the polynomial degree of the shape functions, we examine the convergence of the FE solution for a number of hierarchical grids using (a) the DOF positions and (b) arbitrary locations of assumed data points.