



GPR Full Waveform Sensitivity Analysis using a FDTD Adjoint Method

Giovanni Angelo Meles (1), Stewart Greenhalgh (1,3), Jan van der Kruk (2), Hansruedi Maurer (1), and Alan Green (1)

(1) ETH Zürich, D-ERDW, Zürich, Switzerland (meles@aug.ig.erdw.ethz.ch), (2) Forschungszentrum Juelich GmbH, Juelich, Germany (j.van.der.kruk@fz-juelich.de), (3) Department of Physics, University of Adelaide, Adelaide SA 5005, Australia

Radar tomography is a useful tool for mapping the conductivity and resistivity distributions in the shallow subsurface. These electrical parameters are closely related to important hydrological properties like water content, salinity, porosity and pore structure, and provide information about the clay content and lithological variations within the area of investigation.

Coarse structures involving low electrical contrasts can be profitably imaged by means of cheap and relatively simple methods such as travel time tomography, whereas fine structure involving sub-wavelength detail can only be recovered by inverting full-waveform data. Despite its complexity and high computation costs, full-waveform inversion of GPR data has become a popular tool for high-resolution imaging in engineering and environmental investigations. Most waveform inversion algorithms are based on gradient methods that are less expensive than Gauss-Newton and full-Newton approaches because they do not require the inversion of large matrices in the updating process.

We have recently developed and extensively tested a time-domain full-waveform inversion algorithm that fully takes into account the vectorial nature of the electric field, and performs a simultaneous inversion for conductivity and permittivity rather than in a sequential fashion as with most other schemes. This algorithm enables the inversion of combination crosshole and borehole-to-surface data. Furthermore, we have extended the new inversion scheme by devising a combined frequency-time domain approach, which progressively expands the bandwidth of the data as iterations proceed, starting at low frequency. This new algorithm was designed to tame the non linearity problem which becomes acute when high contrast media are involved. Consequently, it expands the range of applicability of waveform inversion to more realistic scenarios.

A critical aspect of any inversion procedure is the assessment of the reliability and meaning of the final image. Most often, mere convergence in the data space, i.e. the fitting between the observed and the synthetic radargrams, is the only criterion used to appraise the goodness of the final result. A better indication of the correctness of an inverted model, and its various parts, could be obtained by means of a formal resolution analysis. However, whilst gradient-based methods do not involve explicit computation of the sensitivity (or Jacobian) matrix, resolution analysis does. We present here a new method for efficient and explicit calculation of the sensitivities using the FDTD modelling approach. The computation of the Jacobian is performed by means of an adjoint method, which is quite different to brute force and virtual-sources techniques. It provides the values of the Fréchet derivatives (elements of the Jacobian matrix) by cross-correlating forward propagated fields and backward propagated Green's functions. This reduces dramatically the number of forward solutions needed. In fact, while standard methods require $M \times S$ forward runs, where M is the number of model parameters (usually several thousand) and S the number of sources, the adjoint method only needs $S + T$ solutions, where T is the number of receivers (usually a few tens to at most a few hundred forward runs).

We also present here a comparison between model resolution images and cumulative sensitivity images, the latter usually being used as a surrogate for resolution in most practical cases. Although a Gauss-Newton method is not used for the inversion, the information content offered by the radar data can be assessed by eigenvalue analyses of the waveform sensitivity matrix. This allows estimation of the extent of the resolved model space and the unresolved null space, and comparison of the effectiveness between different recording geometries.