



A benchmark comparison of numerical topography: what are suitable sticky-air parameters?

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Topography is a direct observable of the interaction between the Earth's internal and external dynamics. Therefore, it is important for numerical models of lithospheric deformation to compute topography accurately. Earth's surface is a so-called free surface, which means that both normal and shear stress should vanish at this interface. It has been shown that correct treatment of the Earth's surface as a free surface can have a significant effect in models of lithospheric and mantle dynamics (Zhong et al., 1996, Kaus et al., 2008; Kaus et al., 2010). However, a true free surface is computationally expensive which is why most mantle convection simulations until now treat the surface as a free-slip boundary. For these models, topography is computed directly from normal stresses.

A free surface approximation, the so-called "sticky-air", has recently gained interest in the geodynamic community. This method requires the addition of a fluid layer in the model domain while retaining the computational advantage of a free-slip top boundary. The fluid layer is a proxy for air (or water) and should, therefore, have a near-zero density and a viscosity, which is several orders of magnitude lower than the lithosphere viscosity. The interface between material markers defining the crust and the air has been shown to behave similar to a true free surface (Zaleski and Julien, 1992; Gerya and Yuen, 2003b; Schmeling et al., 2008; Quinquis et al., 2011). Sufficiently small normal stress at the surface is ensured by the physical properties of the weak layer (i.e., the low values for density and viscosity).

We present a theoretical background that provides the physical conditions under which the sticky-air approach is a valid approximation of a true free surface. We evaluate two cases that characterise the evolution of topography on different timescales: (1) relaxation of a sinusoidal perturbation and (2) topography changes above a rising sphere. We quantitatively compare topographies for five different numerical codes (using finite difference and finite element techniques) and for several different topography calculation methods (a. direct calculation of topography from normal stress, b. body-fitting methods allowing for meshing the topography, c. Lagrangian tracking of the topography on an Eulerian grid). It is found that the sticky air approach works fine as long as the term $(\eta_{st}/\eta_{ch})/(h_{st}/h_{ch})^3$ is sufficiently small, where η_{st} and h_{st} are the viscosity and thickness of the sticky air layer, and η_{ch} and h_{ch} are the characteristic viscosity and length scale of the model, respectively.

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