



Estimation of Mutual Information from Maximum Entropy distributions and its application to geophysical time series

Carlos Pires

Universidade de Lisboa, Instituto Dom Luiz - Faculdade de Ciencias - DEGGE, Lisboa, Portugal (capires@fc.ul.pt, +351 217500807)

Bivariate Mutual information (MI) reduces to a single growing function ($-\frac{1}{2} \log(1-\rho^2)$) of the absolute correlation ρ when both variables are jointly Gaussian. For non-Gaussian joint distributions there is an extra non-Gaussian MI beyond the Gaussian term. Here, the non-Gaussian MI is estimated from Maximum-Entropy (ME) marginal and joint pdfs, constrained by sets of single and cross moments up to a maximum joint even integer order p ($p=2,6,8,10$), leading to generalized bivariate Gaussians. This is the minimum MI compatible with the imposed expectations. The non-Gaussian MI increases with p with tendency for saturation. An outlier-resistant estimator of the Non-Gaussian MI is obtained by performing previous marginal Gaussian anamorphosis of each variable which are then rotated in order to get uncorrelated new ones. Within this context, we evaluate the effect of high-order joint cumulants in the Non-Gaussian MI. The ME-pdfs belong to the exponential family determined by a set of Lagrange multipliers. These multipliers are found at the minimum of an appropriate functional obtained by a gradient-descent method. We discuss strategies of accelerating convergence and preventing large number floating point during the optimization.

The method is applied to typical time series such as those from deterministic chaos, stochastic processes with additive, multiplicative and power-dependent noises as well as to observed geophysical time series. For that we have used a collection of atmospheric-oceanic indexes for which we evaluate the Gaussian and non-Gaussian time-delay mutual information.

The method appears suitable to diagnose poorly correlated bivariate data in which some non-linear relationship is present and only some cumulants are statistically non-vanishing.