



## Multifractal cascades and the emergence of atmospheric dynamics

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Ordinary fluid flows involve such huge numbers of particles that rather than applying statistical mechanics, one typically exploits emergent macroscopic continuum properties and corresponding higher-level continuum and thermodynamic equations. In a similar way, one expects new higher-level laws to emerge from the chaos of sufficiently strong hydrodynamic turbulence. While the latter presumably continue to obey continuum mechanics, these become impractical and one searches for the emergence of even higher-level laws of “fully developed turuboence”; this was the quest of many of the pioneers of classical turbulence including L.F. Richardson, A. N. Kolmogorov, A. Obukhov, S. Corrsin, and R. Bolgiano.

While these classical laws were often successful in capturing basic scaling properties, a key obstacle was turbulent intermittency. Starting in the 1960's, attempts to deal with intermittency lead to the development of explicit multiplicative cascade models, a subject to which B. Mandelbrot made several important contributions. By the 1980's it became clear that these cascades were the generic multifractal processes and isotropic cascades were applied widely to laboratory turbulence. These applications stimulated the development of multifractal data analysis and modelling techniques; these have subsequently been applied throughout geophysics.

While the atmosphere provides a strongly turbulent natural laboratory and while it differs from dry incompressible hydrodynamics, we nevertheless expect higher-level laws to emerge; indeed, starting twenty-five years ago, cascades were applied to small scale atmospheric turbulence. However, to go to large scales requires new anisotropic notions of scale invariance that can handle cascade processes involving scale by scale stratification in both the vertical plane and in space-time; such “generalized scale invariance” was developed from the mid 1980's, onwards. Atmospheric applications also require large, high-quality - preferably global scale - data sets; the systematic analysis of such planetary scale data (including reanalyses, satellite radiances and aircraft data) has primarily occurred in the last 5 years.

In this review, we show how these theoretical and empirical advances allow us to obtain emergent laws of atmospheric dynamics including waves. Empirically, we show that they apply from milliseconds to decades, from millimetres to the size of the planet. We discuss some consequences including universal properties, nonclassical extreme values and their potential for applications to stochastic parameterisation of numerical models and to stochastic forecasting.