



## Can wind lidars measure turbulence?

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Wind energy has expanded rapidly for several decades and every year thousands of multi-megawatt wind turbines are being installed all over the world. The importance of wind speed measurements can never be overstated since the power produced from the wind turbine is directly proportional to the cube of the wind speed, at least below turbine rated wind speeds. Atmospheric turbulence is one of the main inputs in assessing loads on the wind turbines. Thus, accurate estimation of wind speed and turbulence at several heights is crucial for the successful development of a wind farm. In wind energy the current standard is the use of meteorological masts equipped with cup/sonic anemometers. However, tall meteorological masts are very expensive, and offshore, the costs increase significantly. The advent of remote sensing devices like lidars gives a further boost to the development of wind energy. We use two methods to answer the question posed in the title by developing a theoretical model to estimate the systematic errors in the second-order moments measured by lidars:

1. Conical scanning and velocity azimuth display (VAD) technique to process the data [1] - In this method, the lidar shoots the laser beam at a certain half opening angle  $\phi$  (usually  $30^\circ$  from the vertical) and scans a complete circle. We thus have measurements of radial velocities  $v_r$  at different azimuth angles  $\theta$  along the circle.  $v_r$  is given as a dot product of the unit directional vector and instantaneous velocity field  $\mathbf{v} = (u, v, w)$ , where  $u, v$  and  $w$  are the components of the wind velocity in three directions. The second-order moments are estimated by combining the measurements of  $v_r$  at different values of  $\theta$ . This results in large systematic errors due to the volume averaging along the line-of-sight and the conical section. For this method, we derive the following general lidar equation for the second-order moments,

$$\langle v'_m v'_n \rangle_{lidar} = \int \Phi_{ij}(k) X_i^m(k) X_j^{*n}(k) dk, \quad (1)$$

where  $\int dk \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 dk_2 dk_3$ ,  $k = (k_1, k_2, k_3)$  denotes the wave vector,  $\Phi_{ij}(k)$  is the spectral velocity tensor,  $X_i$  is the weighting function based on the characteristic on the instrument,  $*$  denotes complex conjugation,  $'$  denotes the fluctuations and  $\langle \rangle$  denotes ensemble averaging. Thus, the measurement of the second-order moment by lidar involves interaction of all components of  $\Phi_{ij}(k)$  weighted by the corresponding weighting functions  $X_i^m(k)$  and  $X_j^{*n}(k)$ . It is to be noted that Eqn. (1) is given in Einstein summation convention, and hence, in order to explicitly see the contribution of all components of  $\Phi_{ij}(k)$  on the measurement of the second-order moments by lidar, this equation must be expanded for all values of the subscripts  $i$  and  $j$  (i.e. 1..3). In most cases, the second-order moments are attenuated, whereas in some cases they are also amplified. To compare our theoretical models with the measurements, we use two commercially available wind lidars, the ZephIR as a continuous wave (CW) and the WindCube as a pulsed lidar.

2. Optimum six beam configuration and variances of the radial velocities - In this method, we directly use the variances of  $v_r$  given as [2],

$$\begin{aligned} \langle v_r'^2 \rangle &= \langle u'^2 \rangle \sin^2 \phi \cos^2 \theta + \langle v'^2 \rangle \sin^2 \phi \sin^2 \theta + \langle w'^2 \rangle \cos^2 \phi \\ &+ 2\langle u'v' \rangle \sin^2 \phi \sin \theta \cos \theta + 2\langle u'w' \rangle \sin \phi \cos \phi \cos \theta + 2\langle v'w' \rangle \sin \phi \cos \phi \sin \theta \end{aligned} \quad (2)$$

where  $\langle v_r'^2 \rangle$  is the radial velocity variance, expressed in terms of six second-order moments. Thus, the only systematic errors that arise in the measurement of the second-order moments are due to the volume averaging along the line-of-sight. Since Eq. (2) contains six unknowns, the lidar should scan at least six  $\theta$ s. Scanning the circle at only one  $\phi$ , we get a degenerate solution. We are then presented with a challenge to find an optimum set of  $\theta, \phi$ . In order to formulate our objective function, we assume that the random errors in the measurement of  $\langle v_r'^2 \rangle$  are equal at all  $\theta, \phi$ . We thus formulate a minimization problem based on the ratio of the sum of the error variances of all second-order moments to the error variance of  $\langle v_r'^2 \rangle$ . We find that the optimum configuration is to scan at five  $\theta$ s on a cone with  $\phi = 45^\circ$  and one vertical beam.

We then theoretically estimate the systematic errors in a similar manner as in method 1, and found that the errors in the  $u$  and  $w$  variance are reduced significantly. In order to compare the theoretical results with the measurements, we will use the ZephIR and WindCube along with a newly developed scan head at Risø DTU, Denmark, that can make customized scans.

We conclude that using method 1, wind lidars cannot be used to measure turbulence precisely, whereas using method 2 there is an improvement in the estimation of the  $u$  and  $w$  variances. Our future work consists of using the measurements of Doppler spectra from the ZephIR directly in order to negate the influence of the line-of-sight averaging.

## References

- [1] A. Sathe, J. Mann, J. Gottschall, and M. Courtney. Can wind lidars measure turbulence? *Accepted for publication in the Journal of Atmospheric and Oceanic Technology*, 2011.
- [2] W L. Eberhard, R E. Cupp, and K R. Healy. Doppler lidar measurements of profiles of turbulence and momentum flux. *Journal of Atmospheric and Oceanic Technology*, 6:809–819, 1989.