



Polynomial chaos approximations for stochastic subsurface flow models

fabio nobile (1), lorenzo tamellini (1), and raul tempone (2)

(1) MOX, Department of Mathematics, Politecnico di Milano, Italy (fabio.nobile@polimi.it), (2) Applied Mathematics and Computational Science, KAUST, Saudi Arabia

We consider stochastic models of subsurface flows and aim at approximating efficiently statistical moments of the state variables.

In recent years new approaches have been proposed based on polynomial approximations of the state variables with respect to random variables describing the stochastic model (Polynomial Chaos expansion). These methods are alternative to more traditional statistical approaches as Monte Carlo and exploit the high regularity that the state variable may have with respect to the input random parameters.

The polynomial approximation can be obtained in several ways: by projecting the residual of the equation onto a polynomial subspace (Galerkin procedure); by suitably interpolating the solution in a set of cleverly chosen points in the parameter space (Collocation procedure) or by performing a regression of randomly computed solutions onto a polynomial space (regression procedure). Each of these procedures has pros and cons; whichever is chosen, however, a major question arises on what is the correct polynomial space to use. Indeed, the dimension of the polynomial space grows very fast with the number of random variables employed (curse of dimensionality) and a clever choice of the space is needed to obtain an efficient method, competitive or possibly superior to Monte Carlo.

In this talk we discuss the optimal choice of polynomial spaces by carefully analyzing the dependence of the solution on the input random variables. We will focus mainly on collocation type techniques based on sparse grids of Gauss points and show their effectiveness on some test cases.

References

- [1] I. Babuška, F. Nobile, and R. Tempone. A stochastic collocation method for elliptic partial differential equations with random input data. *SIAM Review*, 52(2):317–355, June 2010.
- [2] J. Bäck, F. Nobile, L. Tamellini, and R. Tempone. Implementation of optimal Galerkin and collocation approximations of PDEs with random coefficients. *Proceedings of CANUM 2010*. To appear in *ESAIM Proceeding*.
- [3] J. Bäck, F. Nobile, L. Tamellini, and R. Tempone. Stochastic spectral Galerkin and collocation methods for PDEs with random coefficients: a numerical comparison. In J.S. Hesthaven and E.M. Ronquist, editors, *Spectral and High Order Methods for Partial Differential Equations*, volume 76 of *Lecture Notes in Computational Science and Engineering*, pages 43–62. Springer, 2011.