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Cassini's angles and periods of free librations of synchronous satellites in Solar system for their rigid nonspherical models

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In the paper the rotations of synchronous satellites of the Earth, Mars, Jupiter, Saturn, Uran and Neptune are studied. On the base theory of resonant rotation of the rigid satellite on precessing elliptical orbit in Andoyer's variables [1] the angles of Cassini and periods of free resonant librations satellites have been determined for big grope of satellites of planets. For the Moon, for Phobos and Deimos, for 8 satellites of Jupiter: Io, Europa, Ganymede, Callisto, Amalthea, Phebe, Metis, Adrastea; for 11 satellites of Saturn: Titan, Rhea, Diona, Iapetus , Tethys, Enceladus, Mimas, Janus, Epimetheus, Atlas, Telesto; for 12 satellites of Uran: Titania, Oberon, Umbriel, Arial, Miranda, Portia, Juliet, Belinda, Cressida, Desdemona, Bianca, Ophelia and for 7 satellites of Neptune: Triton, Proteus, Larissa, Galatea, Despina, Thalassa, Naiad (all 41 satellites).

The orbital motion of satellites have been described by mean elements of the perturbed orbital motions according to data (http://ssd.jpl.nasa.gov/sat_elem. Html) (http://ssd.jpl.nasa.gov/?sat_elem.). From them: the eccentricity (e), the inclination of orbit plane (i), the mean orbital motion and its period (n and T_n), the angular velocity and period of preseccion of orbit plane of satellite on local Laplacian plane (n_Ω and T_Ω). In our approach all mentioned parameters are considered as constants and more fine effects in orbital motions of satellites do not take into account in this paper. So, orbital motions of satellites is to uniformly precessing orbits with a constant angle of inclination i relatively to the base plane of coordinate system (the local Laplace plane or equatorial plane of the planet, etc.) and at a constant rate of precession $n_\Omega < 0$. It is important to emphasize that for the Moon we have used mean ecliptic orbital elements for epoch 2000 Jan. 1.50 TT (base solution: DE405/LE405). For Phobos and Deimos - the mean orbital elements referred to the local Laplace planes. Epoch 1950 Jan. 1.00 TT (solution: MAR080). For Jupiter satellites we have been used mean orbital elements referred to the local Laplace planes. Epoch 1997 Jan. 16.00 TT (JUP230 - JPL satellite ephemeris) http://ssd.jpl.nasa.gov/sat_elem. For Saturn satellites the mean orbital elements referred to the local Laplace planes (epoch 2000 Jan. 1.50. TT) have been used. For the satellites of Uran mean equatorial orbital elements (epoch 1980 Jan. 1.0 TT. Solution: GUST86) have been used. And for Neptune satellites we have used the mean orbital elements referred to the local Laplace planes (epoch 2000 Jan. 1.50 TDT, Voyager, HST, and Astrometric data solution).

In our studying of the Cassini's motions of these satellites the model values of coefficients of second harmonics of gravitational potentials (J_2 and C_{22}) and of dimension less moment of inertia $I = C/\left(mr^2\right)$ were used. Here C is the polar moment of inertia, m and r is the mass and the mean radius of satellite. For 4 satellites of Jupiter and for 2 satellites of Saturn: Io, Europa, Ganimede, Callisto and Rhea and Titan the values of mentioned parameters, obtained on the base of data of space missions to these bodies have been used. For a wide list of other satellites these parameters were evaluated on the basis of available data on their models as homogeneous ellipsoids with axes: $D_a > D_b > D_c$by the simple formulas:

$$J_2 = \frac{D_a^2 + D_b^2 - 2D_c^2}{40r_m^2}, \quad C_{22} = \frac{D_a^2 - D_b^2}{80r_m^2}, \quad I = \frac{D_a^2 + D_b^2}{20r_m^2}, \quad r_m = \sqrt[3]{D_a D_b D_c/8}.$$

Here D_c is a polar axis of ellipsoid, D_a is a axis of sinchronous ellipsoid directed to the mother planet. $r_m$is a model mean radius of satellite, which is determined as the radius of a sphere with volume equal to the volume of an ellipsoid with axes $D_a > D_b > D_c$. Such models have been built earlier in [1], [2] for a wide variety of satellites of solar system planets. In this paper, a list of such models is increasing, and parameters of models I, J_2, $C_{22}$are specified. These models also have obtained here effective applications.

For 4 satellites of Saturn and Uranus (Diona and Titania, Oberon, Umbriel) (for which do not have sufficient data about ellipsoidal form and sizes) we have been used ellipsoidal model of hydrostatic equilibrium on the Goldreich, Peale results [3]. The full list of discussed parameters for satellites of planets is presented in the paper [1]. For these satellites we use conditional value of their dimension less moment of inertia $I = 0.35$. For some satellite parameters J_2, $C_{22}$and I have been obtained by different methods and as it was shown hydrostatic models, models of homogeneous ellipsoids and space mission models generally are in good agreement.

The purpose of paper is to study syncronous motions of satellites in Solar system and for each of them to determine the value of the basic Cassini's parameter ρ_0 (it is the average angle of inclination of the axis of rotation relatively to normal of the precessing orbit plane) and the periods of resonant librations in the longitude (T_g), in the pole wobble (T_l) and period of space precession (T_h) (and their errors). Here we use the analytical formulas for mentioned parameters which were developed by study of the Moon Cassini's motion in my early papers (Barkin, 1978, 1979) [1].

Table 1. Values of Cassini's angleρ_0 for all synchronous satellites of the planets with$\rho_0 \geq \ 0^0 001$.

Satellites	ρ_0	i	$i + \rho_0$	$i + \rho_0$
Moon	**6º782**	**5º16**	**1º622***	**5839"2***
Triton	**0º214**	**156º9**	**157º1**	**-22º911**
Titan	**0º096**	**0º312**	**0º408**	**1468"8**
Mimas	**0º095**	**1º574**	**1º670**	**6012"0**
Europa	**0º055**	**0º466**	**0º521**	**1875"6**
Oberon	**0º040**	**0º068**	**0º108**	**388"8**
Tethys	**0º032**	**1º091**	**1º123**	**4042"8**
Ganymede	**0º031**	**0º177**	**0º208**	**748"8**
Rhea	**0º029**	**0º333**	**0º362**	**1303"2**
Miranda	**0º020**	**4º338**	**4º358**	**15688"8**
Callisto	**0º012**	**0º192**	**0º204**	**734"4**
Titania	**0º009**	**0º079**	**0º088**	**316"8**
Naiad	**0º009**	**4º691**	**4º700**	**16920"0**
Iapetus	**0º007**	**8º313**	**8º320**	**29952"0**
Phebe	**0º006**	**1º080**	**1º086**	**3909"6**
Epimetheus	**0º004**	**0º351**	**0º355**	**1278"0**
Telesto	**0º003**	**1º180**	**1º183**	**4258"8**
Io	**0º002**	**0º036**	**0º038**	**136"8**
Amalthea	**0º002**	**0º380**	**0º382**	**1375"2**
Janus	**0º002**	**0º163**	**0º165**	**594"0**
Umbriel	**0º002**	**0º128**	**0º130**	**468"0**
Adrastea	**0º001**	**0º054**	**0º055**	**198"0**
Diona	**0º001**	**0º028**	**0º029**	**104"4**
Arial	**0º001**	**0º041**	**0º042**	**151"2**

*** for the Moon here is a difference between the angles ρ_0 and i**

Specially for the case of small eccentricities and inclinations of orbits of synchronous satellites we have obtained the simple reduced formulas for all four considered parameters. The small value of Cassini's angle ρ_0 and corresponding periods of free librations of satelltes are determined by the grope of knowm (but approximate) formulae [1], [2]:

$$\rho_0 = -\sin i / (\cos i + J), \quad J = \frac{3n}{4n_\Omega I}\left[\left(2 + 3e^2\right) J_2 + 2\left(2 - 5e^2\right) C_{22}\right]. \tag{1}$$

$$T_l = IT_n \Big/ \left[2\left(J_2^2 - 4C_{22}^2\right)^{1/2}\right], \quad T_g = T_n \Big/ \left[2\left(3C_{22}/I\right)^{1/2}\right], \quad T_h \approx T_\Omega \left(\sin\rho_0 / \sin i\right). \tag{2}$$

Here we must remark that for all considered satellites in this paper in acordance with generalized Cassini's laws ascending node of the orbit coinside with ascending node of the equator on the local Laplacian plane. Similar situation has place in the Mercury resonant motion.

Table 1 shows the calculated (theoretical) values of Cassini's angles for 24 satellites of the planets in the solar system. Here are selected only those satellites for which the values of the angle between the normal to the average plane of the orbit and the angular velocity of rotation$\rho_0 \geq \ 0^0 001$. Satellites ordered by the values of Cassini's angle (first column), starting from the Moon. It also shows the value of inclinations of orbits of satellites i with respect to the reference plane (to the local Laplace plane or equatorial plane of the planet, or to the ecliptic of the epoch (in the case of the Moon)). Also here the values of summary angles $i + \rho_0$ are given in degrees and in arcseconds.

The report provides a comparative analysis of satellite motions on the Cassini/ The periods of free resonant librations, eccentricities of the trajectories of the poles of the axes of rotation of satellites and others dynamic characteristics have been calculated.

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