



Reconstruction of a dynamical system underlying an observed time series by optimal stochastic models

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The majority of natural systems (climate system including) are known to be both high-dimension and open, i.e. subject to numerous external forcings. Hence in many cases they produce complex multi-scale behavior which cannot be modeled in deterministic way by direct analysis of observed processes. A basic idea underlying the suggested stochastic approach is that the robust dynamic properties of the system evolution can be described by a few variables, while other features may be considered as a stochastic disturbance. Models in a form of random dynamical systems (RDS) present a necessary and important step towards reconstructing the observed systems when their adequate first-principle mathematical models are either unknown or subjected to further verification. Note that, even for deterministic systems, the construction of a deterministic model from the observed time series and use of this model for prediction has quite a number of principal restrictions connected with high embedding dimension and overembedding problem. Reconstruction in the form of RDS (stochastic model) removes these restrictions, thus making the proposed approach more universal.

This report is devoted to developing of an approach to construction of an optimal model reproducing basic dynamical properties of the system underlying observed time series. We represent the evolution operator of investigated system in a form of RDS including deterministic and stochastic parts, both parameterized by artificial neural network. Then we use Bayesian information criterion for estimation of optimal complexity of the model, i.e. both number of parameters and dimension corresponding to most probable model yielding the analyzed process. We demonstrate on the number of model examples that the model with non-uniformly distributed stochastic part (which corresponds to non-Gaussian perturbations of evolution operator) is optimal in general case. Further, we show that simple stochastic model (including a few variables) can be most preferred for reconstruction of the evolution operator underlying complex observed dynamics even in a case of deterministic high-dimensional system. Workability of suggested approach for modeling and prognosis of real-measured geophysical dynamics is investigated.