



## Rock-fracture populations: power laws, entropy, and strain energy

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Rock fractures commonly form systems or networks, such as fissure swarms and fault zones, that evolve through time. In many such systems the fracture lengths and the fracture displacements follow power-law size distributions. Such distributions imply that there are a large number of small events, processes, or objects (here fractures) of a particular type and a small number of large events, processes, or objects of the same type. More specifically, power laws are scale free; in contrast to, say, normal (Gaussian) distributions, there are no objects (or events or processes) that are typical for the distribution as a whole. For a cumulative frequency (probability) distribution, a power law has the form

$$P(\geq x) = Cx^{-\gamma} \quad (1)$$

where  $P(\geq x)$  is the number or frequency of objects (here, the number of fractures) with a characteristic linear dimension (here length) larger than  $x$ ,  $C$  is a constant of proportionality, and  $\gamma$  is the exponent that characterises the relation between size (here fracture length) and number (here, of fractures). The factor  $x$  is also referred to as a linear scaling factor of the objects to the whole. When the logarithms of the values ( $x$ ) are plotted against the logarithm of  $P(x)$ , that is, on a bi-logarithm or a log-log plot, a power law such as Eq. (1) yields a straight line, the slope of which is  $\gamma$ . The exponent  $\gamma$  is variously referred to as such, or as the scaling exponent (or factor), or, sometimes, as the exponent or fractal dimension. Fractal (exponent) dimensions have been used for many years to describe certain geometric relationships that develop during rock fragmentation and fracture development.

Entropy is a well-known concept from thermodynamics, statistical mechanics, and information theory. Entropy for a mechanical system is related to energy principles: an isolated system that develops so as to reach equilibrium with its external and internal forces tends to attain a maximum entropy and a minimum potential energy or, in terms of fracture development, a minimum strain energy. The strain energy of a crustal segment is directly related to the mechanics of fracture development through the Griffith crack theory and its extension to modern rock-fracture mechanics. While entropy and strain energy are thus related, the possible relation of these two with fractal dimensions has received less attention.

This presentation has three aims. The first aim is to present data on rock fractures in Iceland that range in length from a few tens of centimetres to many kilometres, that is, over five orders of magnitude. All the data are from rocks with essentially uniform mechanical properties (thick, Holocene, pahoehoe lava flows) located within the active rift zone of Iceland. The second aim is to show how the entropies and the fractal dimensions of the fracture populations are related. Here the focus is on the entropy calculations for the various subpopulations and how they can be correlated with their fractal dimensions. The third aim is to explore the relation between entropy and strain energy, and how this relation can be used to explain the development of rock-fracture populations.