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Damped Frank-Kamenetskii Approximation for Modelling of Terrestrial Planets with High Surface Temperatures (Venus) or Large Masses (Super-Earths)

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The viscosity of a silicate mantle is strongly dependent on temperature and pressure and can be described by the Arrhenius law [1]. Assuming realistic values of the activation energy and volume, viscosity variations in a terrestrial mantle amount to values of about $10^{10} Pas$ or larger in the upper mantle taking into account elasto-viscous material. Many numerical codes, however, cannot deal with such large viscosity contrasts (especially if the effect of elasticity is ignored and viscosity contrasts increase to much higher values). In general, a smaller viscosity contrast has the advantage of smaller computation times and less numerical errors.
A common method to reduce the viscosity contrast is to use the Frank-Kamenetskii approximation [2]. Applying this approximation for purely temperature-dependent viscosity, the results are similar to those using the Arrhenius law when the convection is in the steady-state stagnant lid regime [3; 4].

Here, we compare the Frank-Kamenetskii approximation to the Arrhenius law for both temperature- and pressure-dependent viscosity. To this end, we have derived new Frank-Kamenetskii parameters that also include the pressure-dependence of the viscosity and differ from previous approximations [5; 6]. We show that using these parameters, the depth-dependence of the approximated viscosity is comparable to the more realistic Arrhenius viscosity. It is even possible to model a stagnant lower mantle that may form above the core-mantle boundary in case of high activation volume or high pressure like for massive Super-Earths [7].

It should be noted, however, that for high surface temperatures as is the case for Venus this approximation does not represent the mantle flow as it is derived using the Arrhenius law. With the classical Frank-Kamenetskii approximation viscosity contrast of less than $\sim 10^5$ develop, i.e. the minimal viscosity contrast necessary to be in the stagnant lid regime is not reached. The convection regime changes to the transitional or mobile regime [3]. To overcome this problem, we have further derived a new approximation, which we call the Damped Frank-Kamenetskii approximation. This is a mixture between the classical law and a second-order approximation controlled by a damping parameter. The second-order approximation is not a linearization of the exponential viscosity term like for the classical Frank-Kamenetskii approximation, but is a quadratic approximation with a higher accuracy. The new method leads to a stagnant lid in all cases treated in our investigations. Note that for planets with low surface temperatures, this new approximation can be used as well with a damping parameter of zero that yields the standard Frank-Kamenetskii approximation.

Our studies suggest that the classical Frank-Kamenetskii approximation with the here derived parameters can be used to simulate the mantle dynamics in Super-Earths even with a high pressure-dependence of the viscosity if the viscosity contrast across the upper thermal boundary is above $\sim 10^5$ allowing a stagnant lid to form on top the convecting mantle. If the constraint of a stagnant lid is not satisfied, e.g. for high surface temperatures, the new Damped Frank-Kamenetskii approximation should be used instead.

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