The relative importance of heat exchange mechanisms in karst conduits

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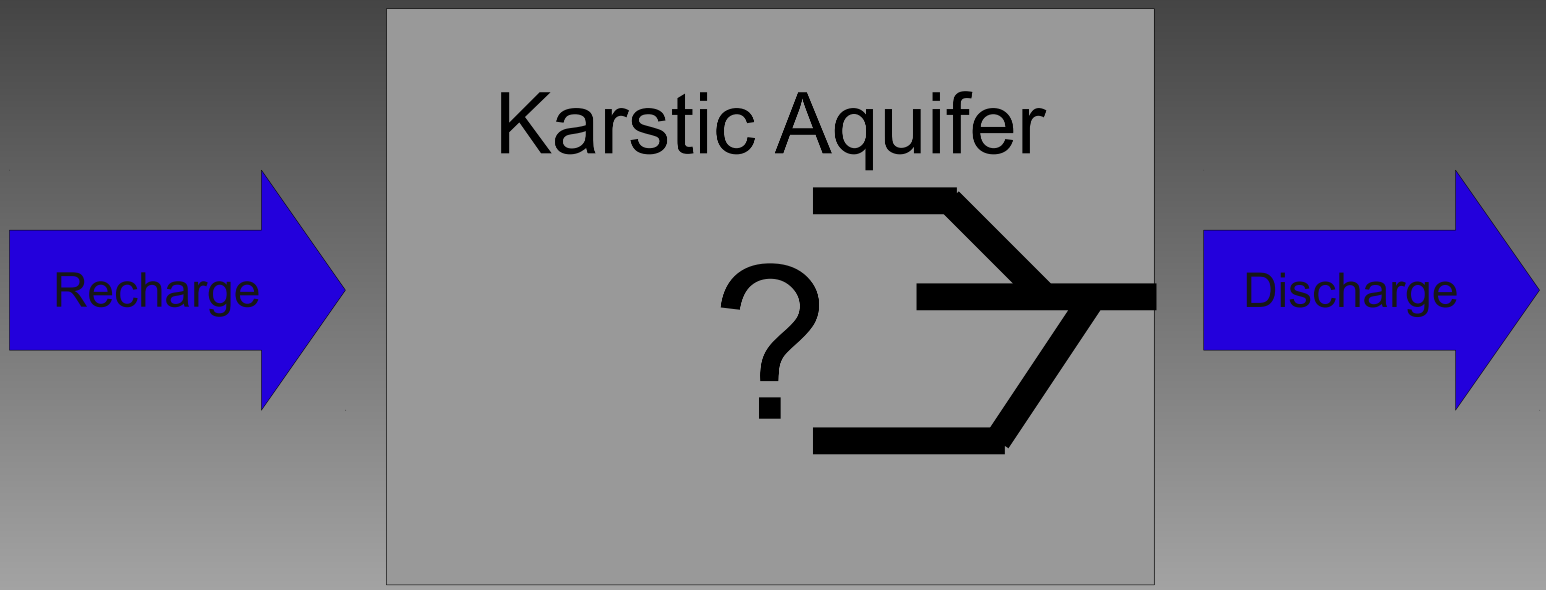
The central problem in karst hydrology?



Conduits largely determine the flow and transport through the system, but in most cases we have incomplete information about the conduit network.

Using Spring Response to Probe Conduit System Geometry

- Analysis of hydrographs/recession curves (many studies)
  Limitations: can be a strong function of recharge (Covington, Wicks, and Saar, 2009, WRR; Mahler, 2009, GSA; Martin and Bailey-Comte, submitted).

- Correlations between thermal and chemical responses and conduit geometry (e.g. Ashton, 1966; Benderitter et al. 1993; Liedl, Renner, and Sauter, 1998; Grasso et al. 2003; Birk et al. 2004, 2006; Luhmann et al., in review)

Equations for Heat Transport in a Karst Conduit

$$\frac{\partial T_{\mathrm{w}}^*}{\partial t^*} = \frac{1}{\mathrm{Pe}} \frac{\partial^2 T_{\mathrm{w}}^*}{\partial x^{*2}} - \frac{V}{\bar{V}} \frac{\partial T_{\mathrm{w}}^*}{\partial x^*} + \mathrm{St}(T_{\mathrm{s}}^* - T_{\mathrm{w}}^*),$$

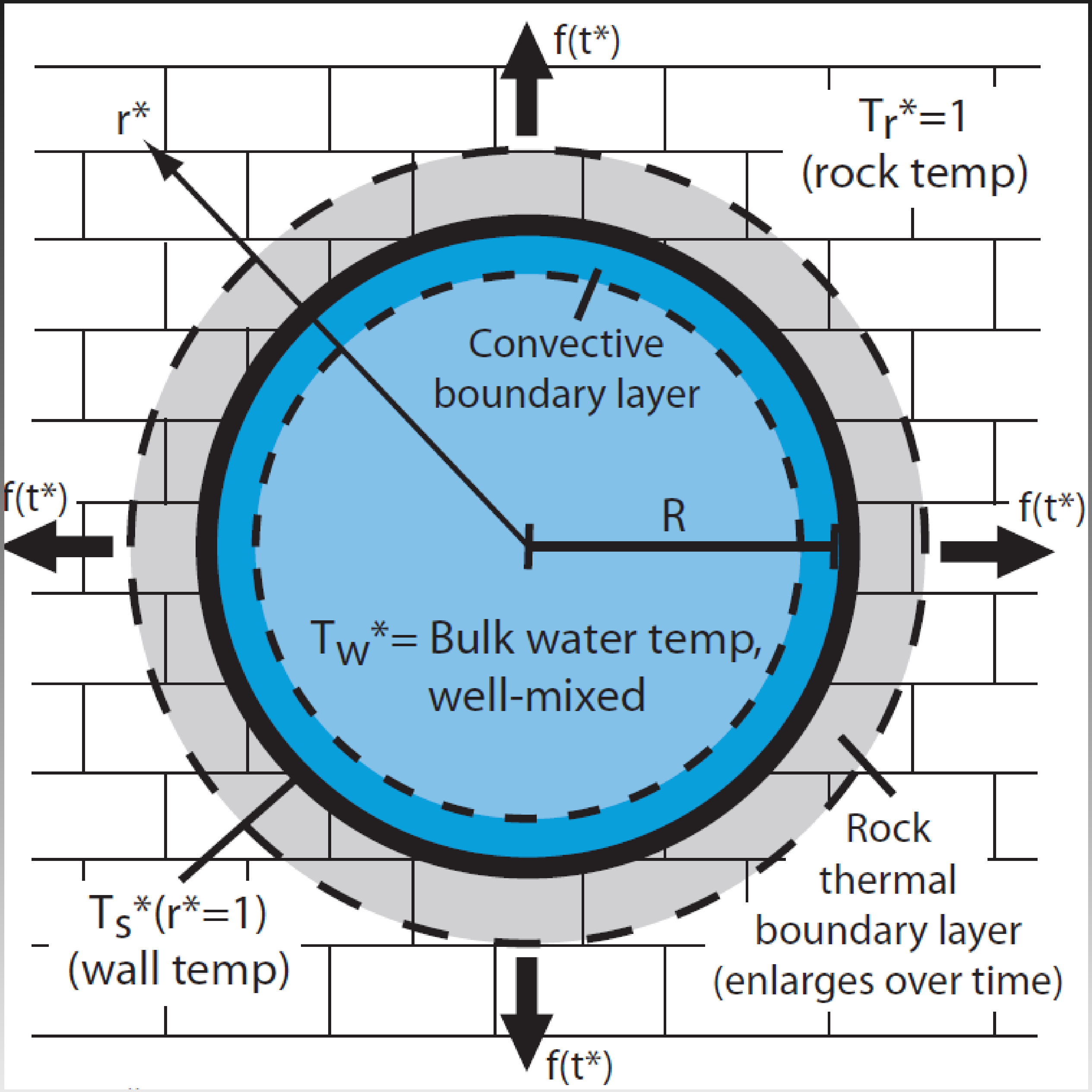
Heat Advection-Dispersion Equation

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_{\mathrm{r}}^*}{\partial r^*} \right) + \frac{R^2}{L^2} \frac{\partial^2 T_{\mathrm{r}}^*}{\partial x^{*2}} = \Theta \frac{\partial T_{\mathrm{r}}^*}{\partial t^*},$$

Cylindrical Heat Conduction Equation

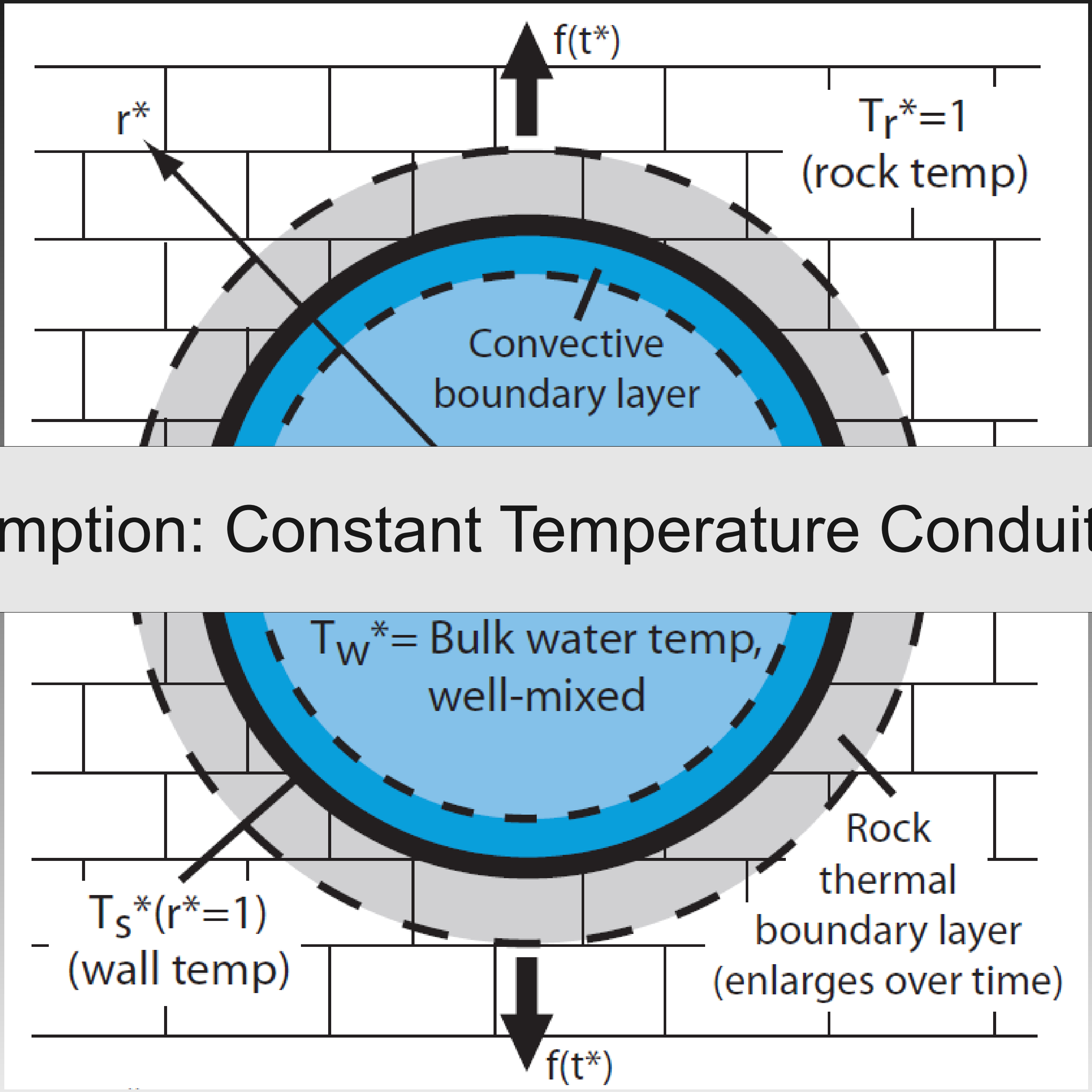
(Covington et al., in review, WRR)

Convection and conduction act in series to control heat exchange



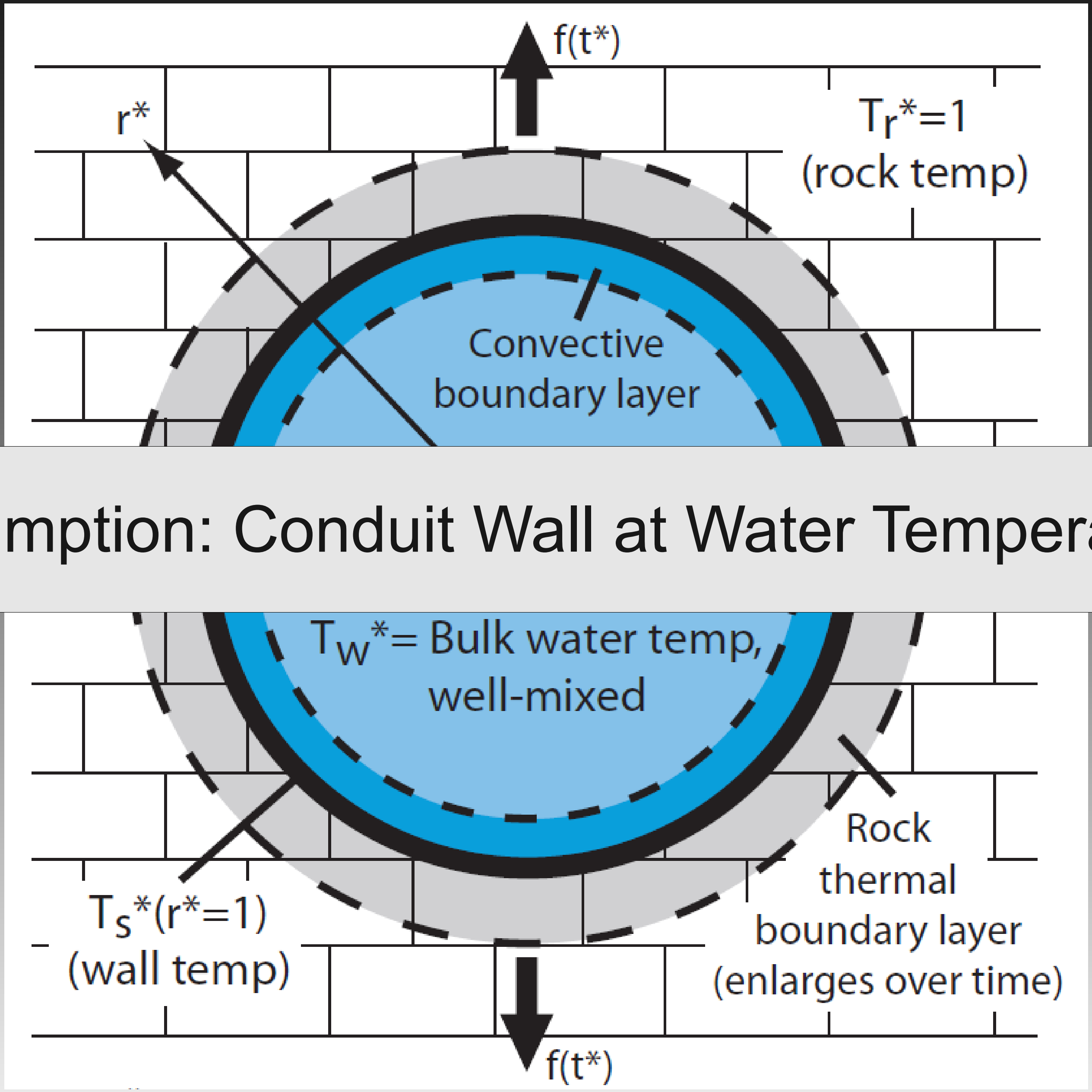
(Covington et al., in review, WRR)

Convection and conduction act in series to control heat exchange



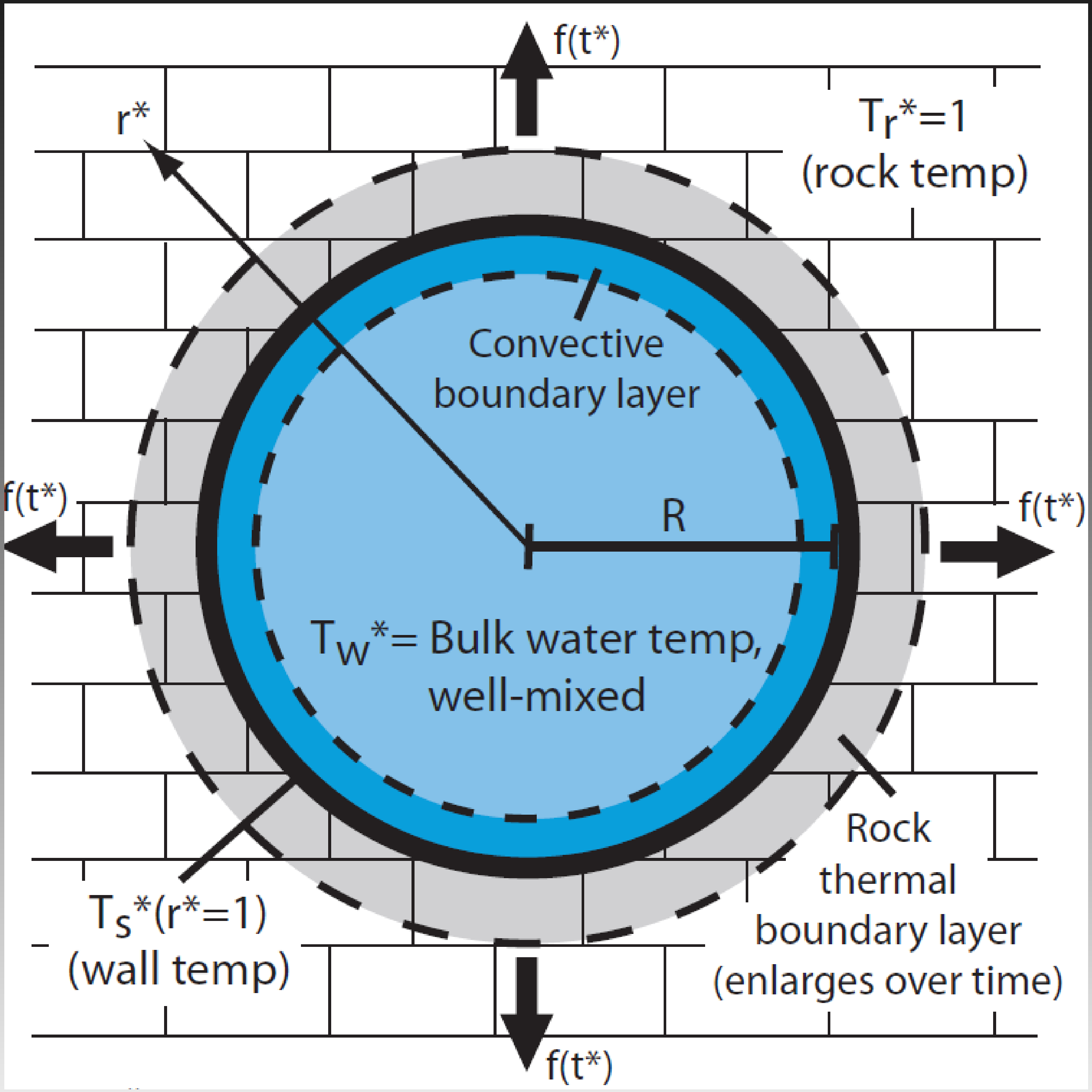
Assumption: Constant Temperature Conduit Wall

Convection and conduction act in series to control heat exchange



Assumption: Conduit Wall at Water Temperature

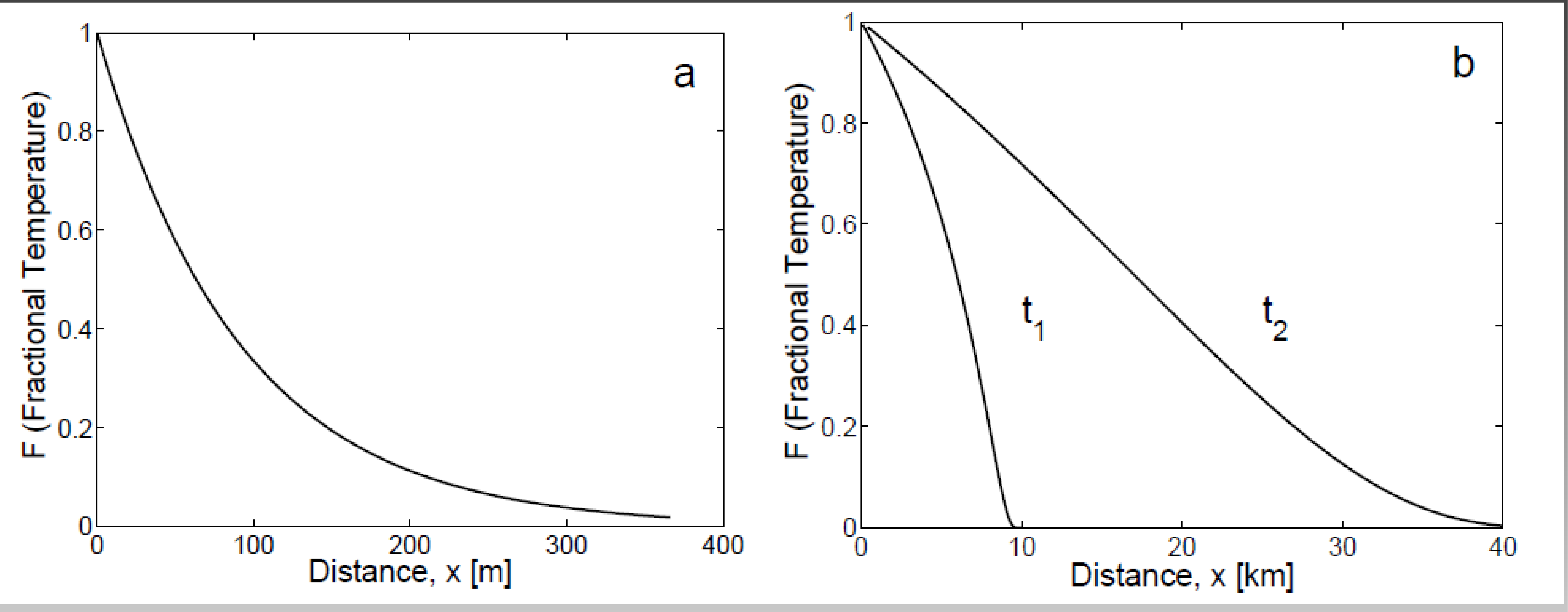
Convection and conduction act in series to control heat exchange



The different approximations produce dramatically different behavior

Convection-limited

Conduction-limited



400 Meters

40 Kilometers

(Covington et al., in review, WRR)

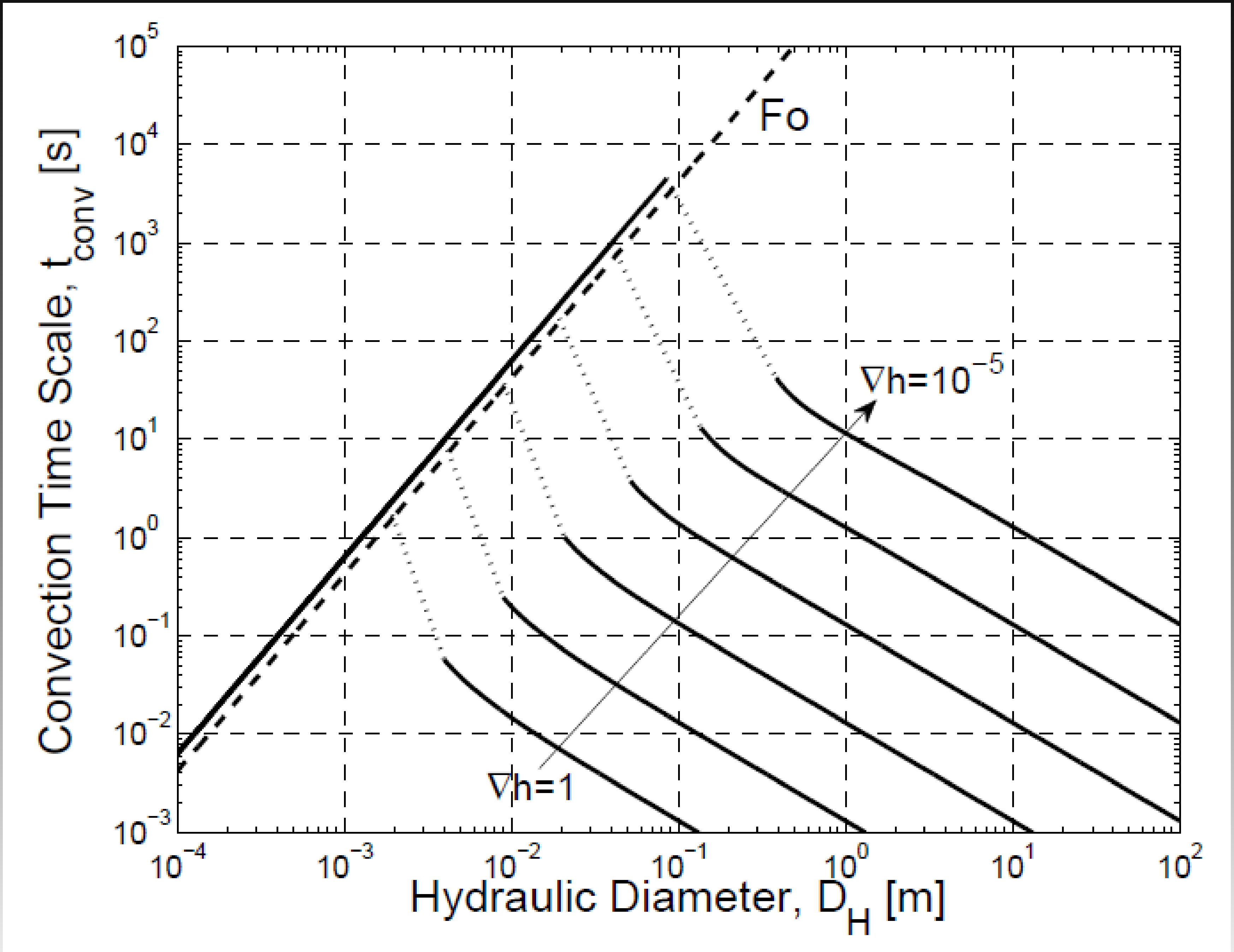
The appropriateness of each solution is determined by a characteristic time scale

- At early times, the skin depth of the heat pulse in the conduit wall is very shallow, and conductive rates are accordingly high, such that the convection-limited solution provides a good approximation

- At late times, the conduit wall approaches the water temperature and the conduction-limited solution is good.

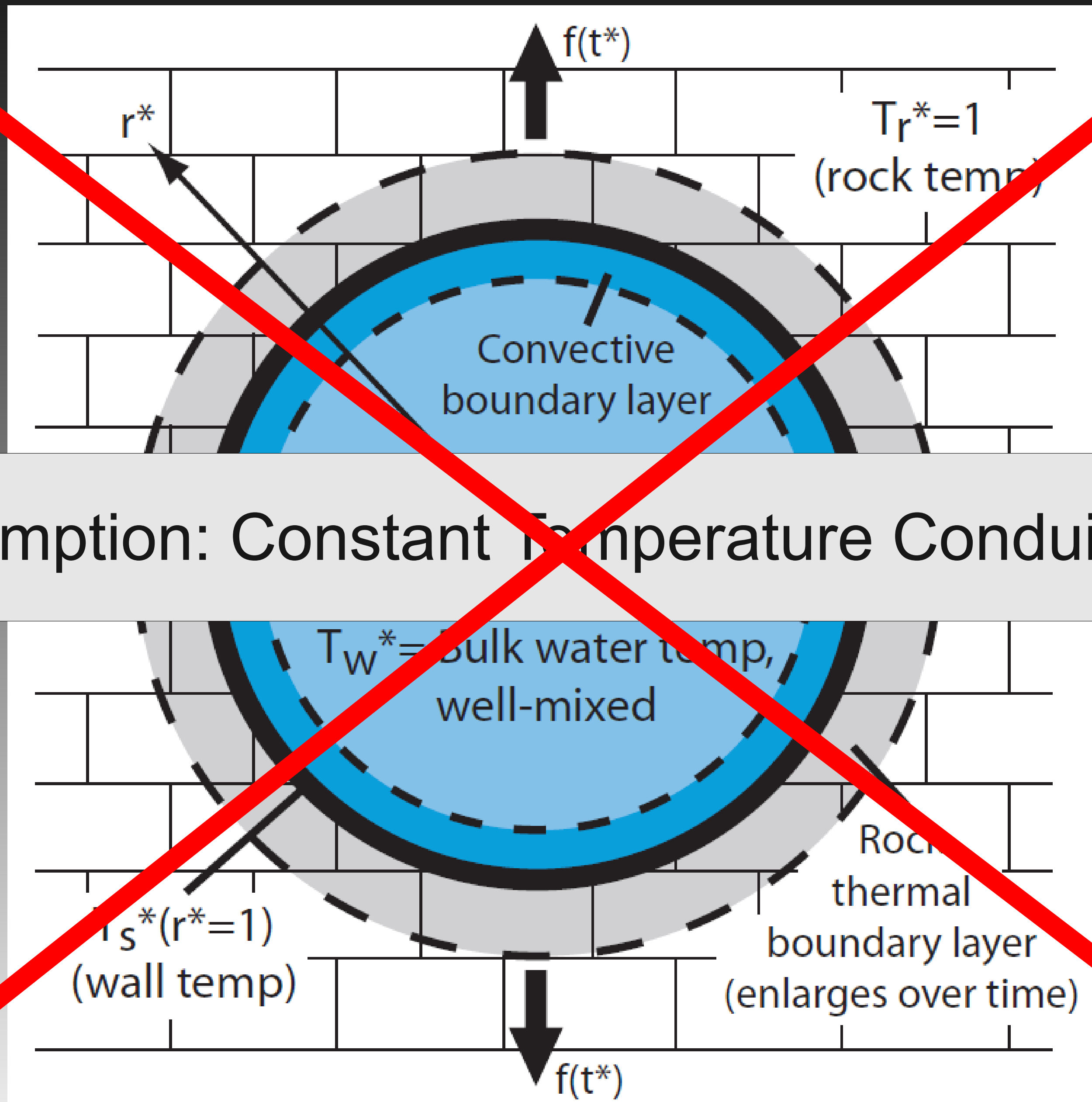
This time scale can be approximated using analytical solutions of a heat pulse propagating into a semi-infinite solid.

Time scale over which conduit wall reaches water temperature



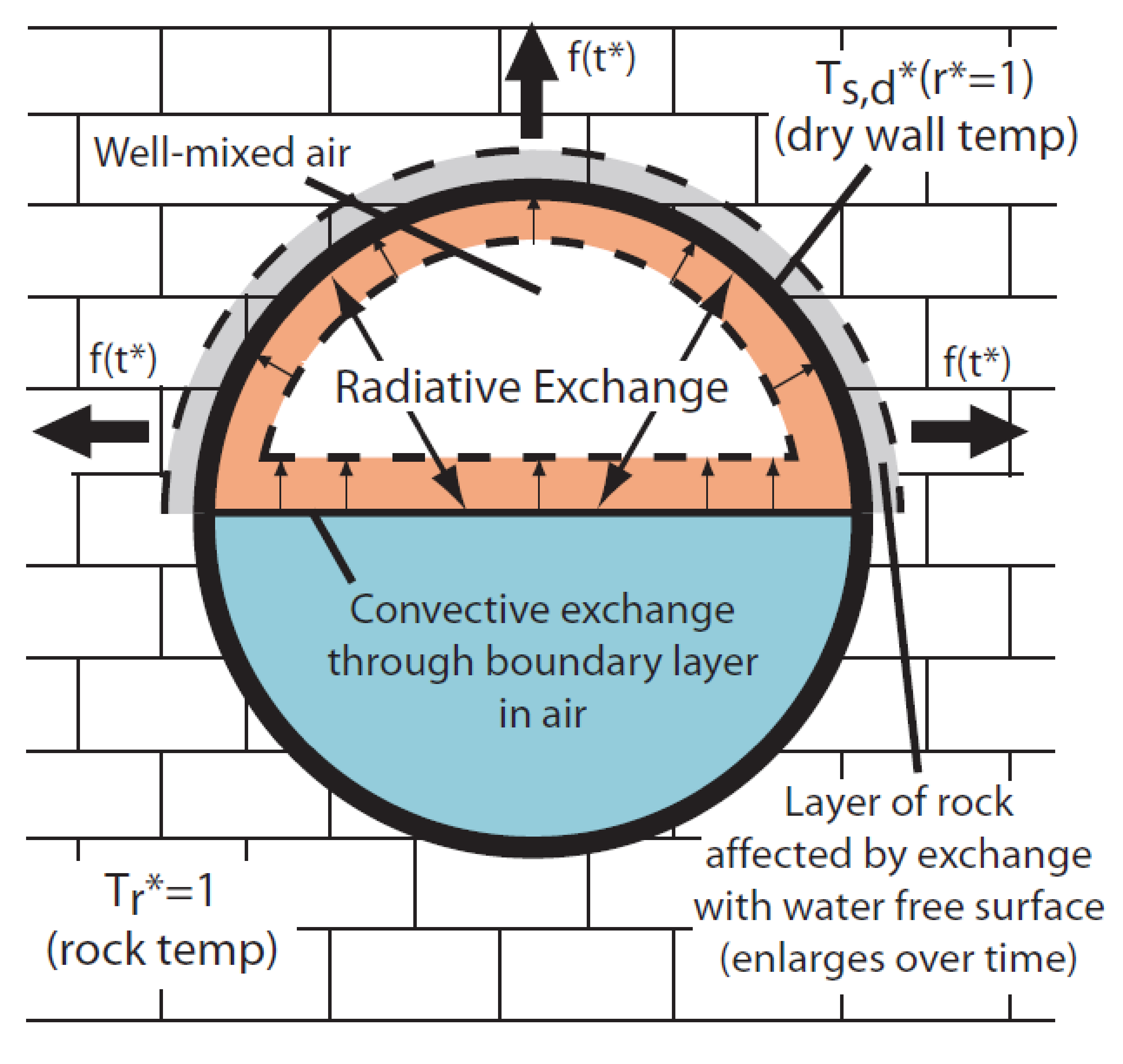
(Covington et al., in review, WRR)

Convection and conduction act in series to control heat exchange



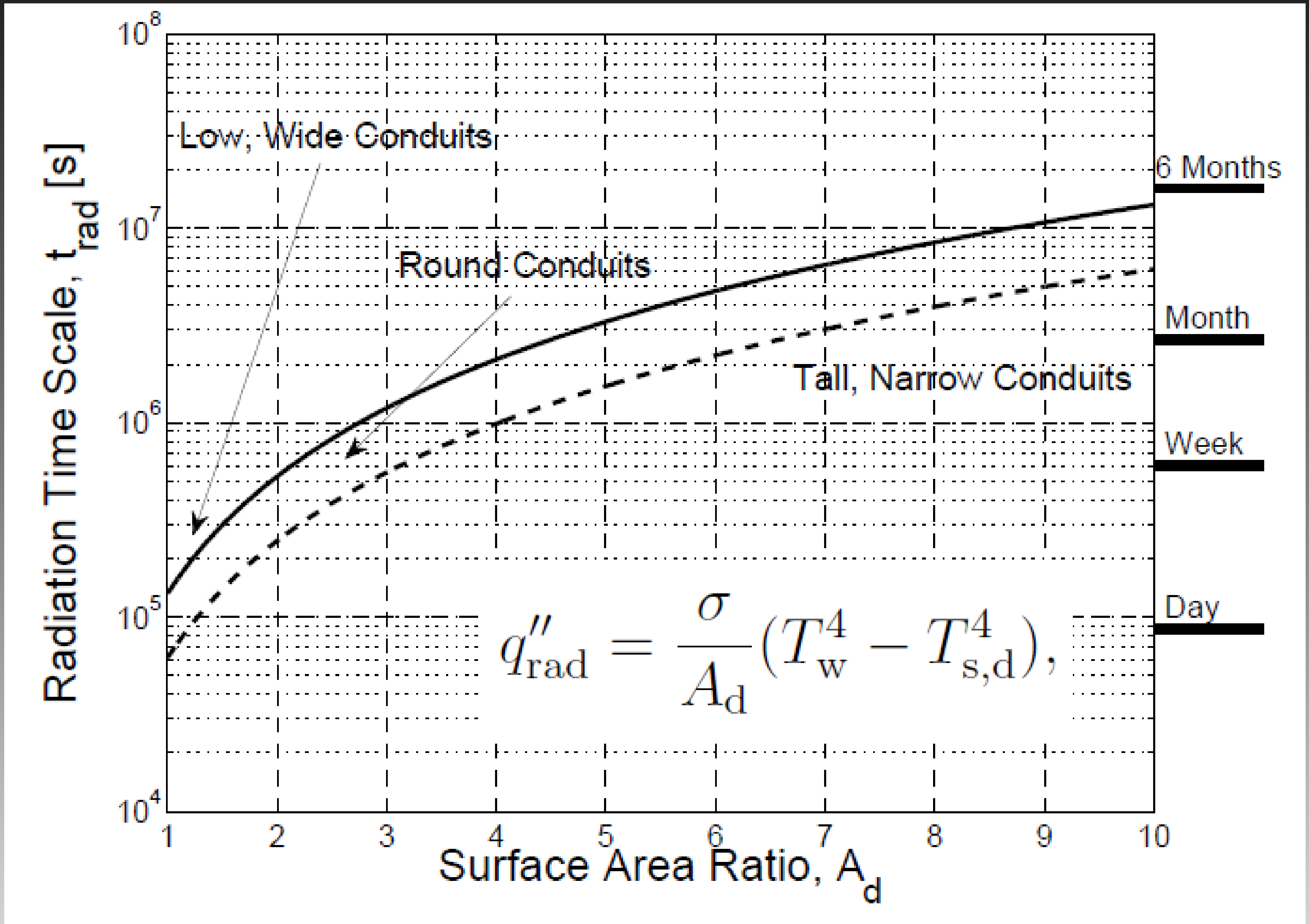
Assumption: Constant Temperature Conduit Wall

What about heat exchange in open channel conduits?



(Covington et al., in review, WRR)

Time scale over which conduit wall reaches water temperature via radiative exchange

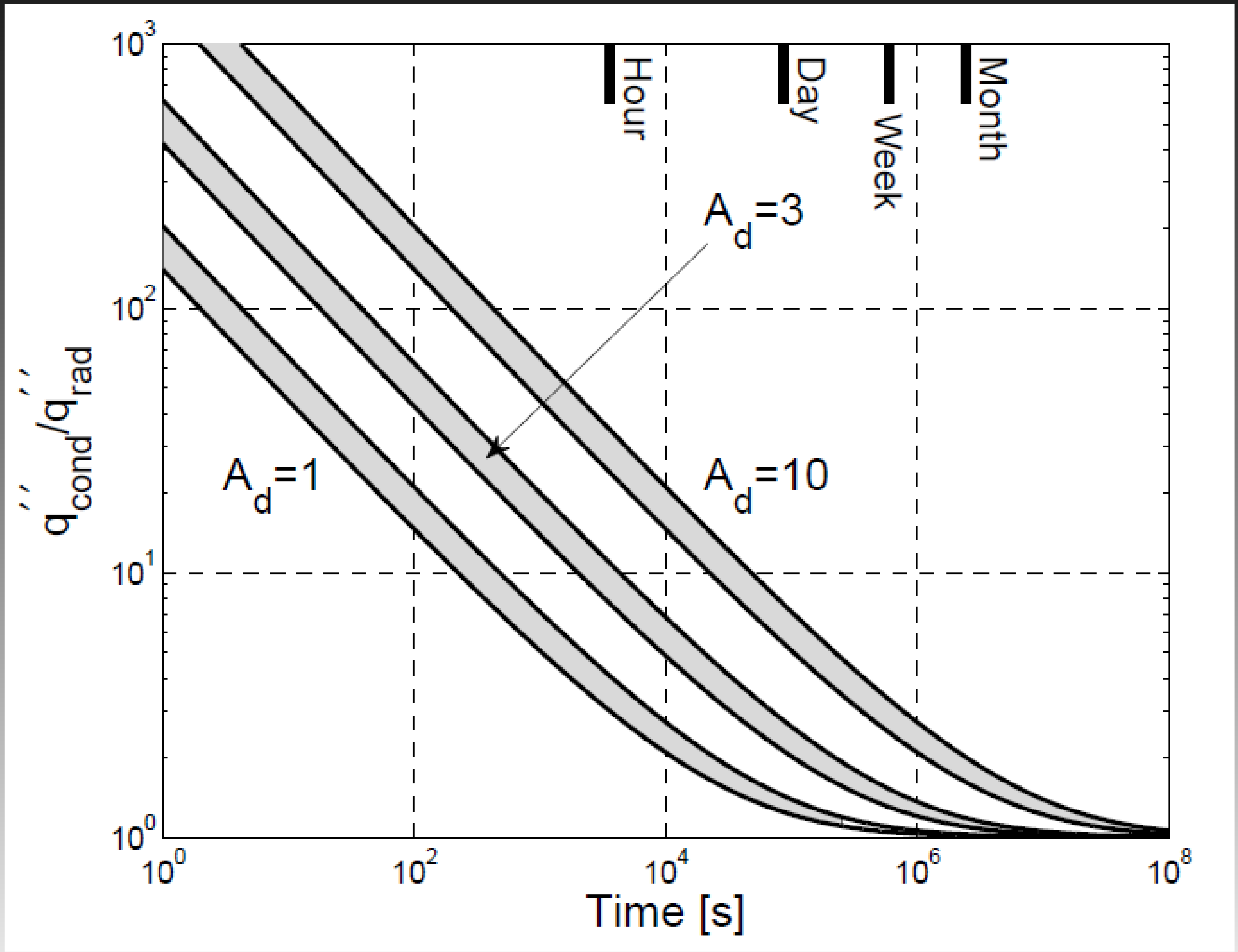


$$A_{\mathrm{d}} = P_{\mathrm{d}}/W_{\mathrm{fs}}$$

P_d = conduit dry perimeter,

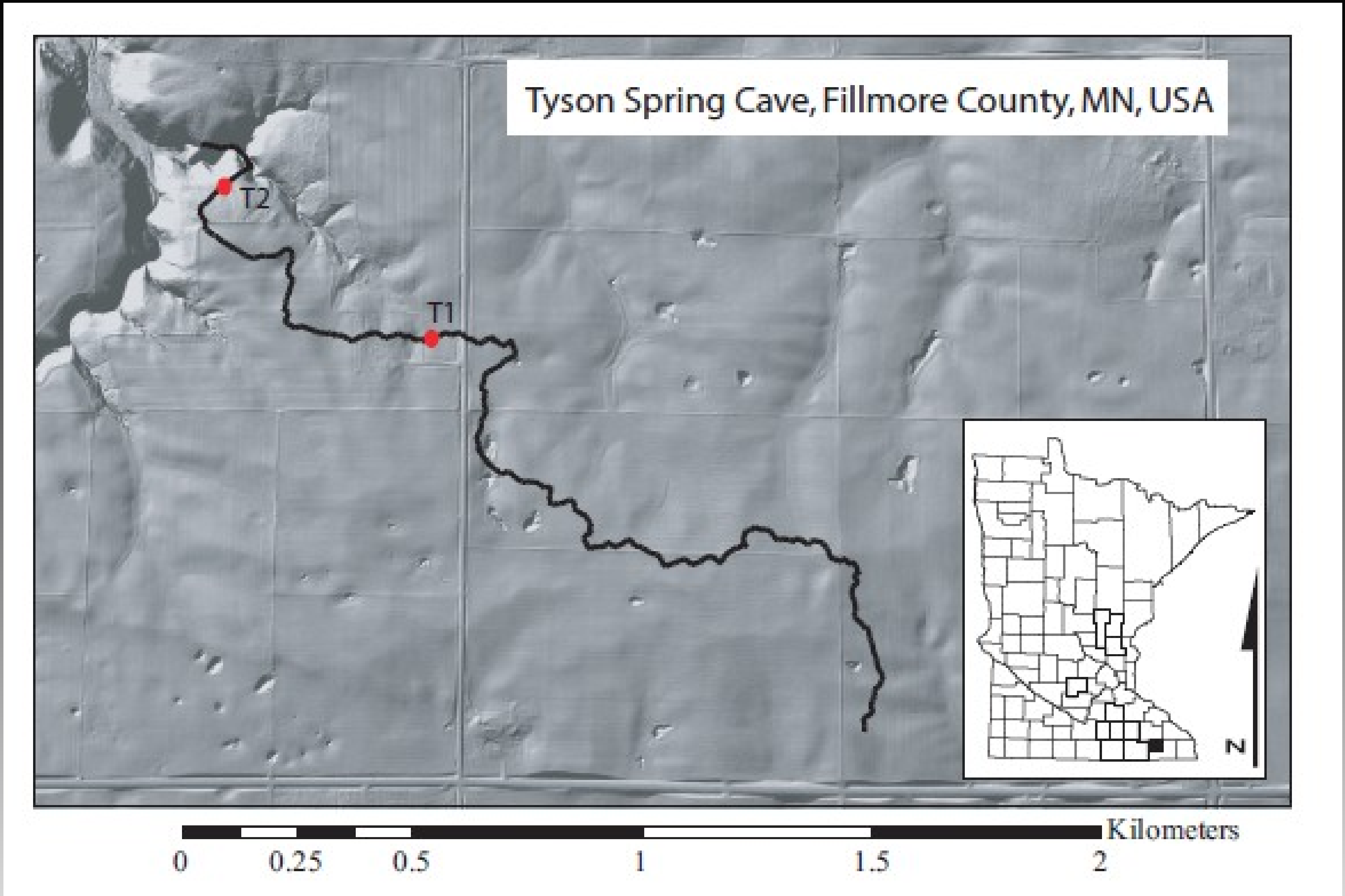
W_{fs} = width of water free surface

Ratio of heat fluxes at the water/rock and air/rock interfaces as a function of time



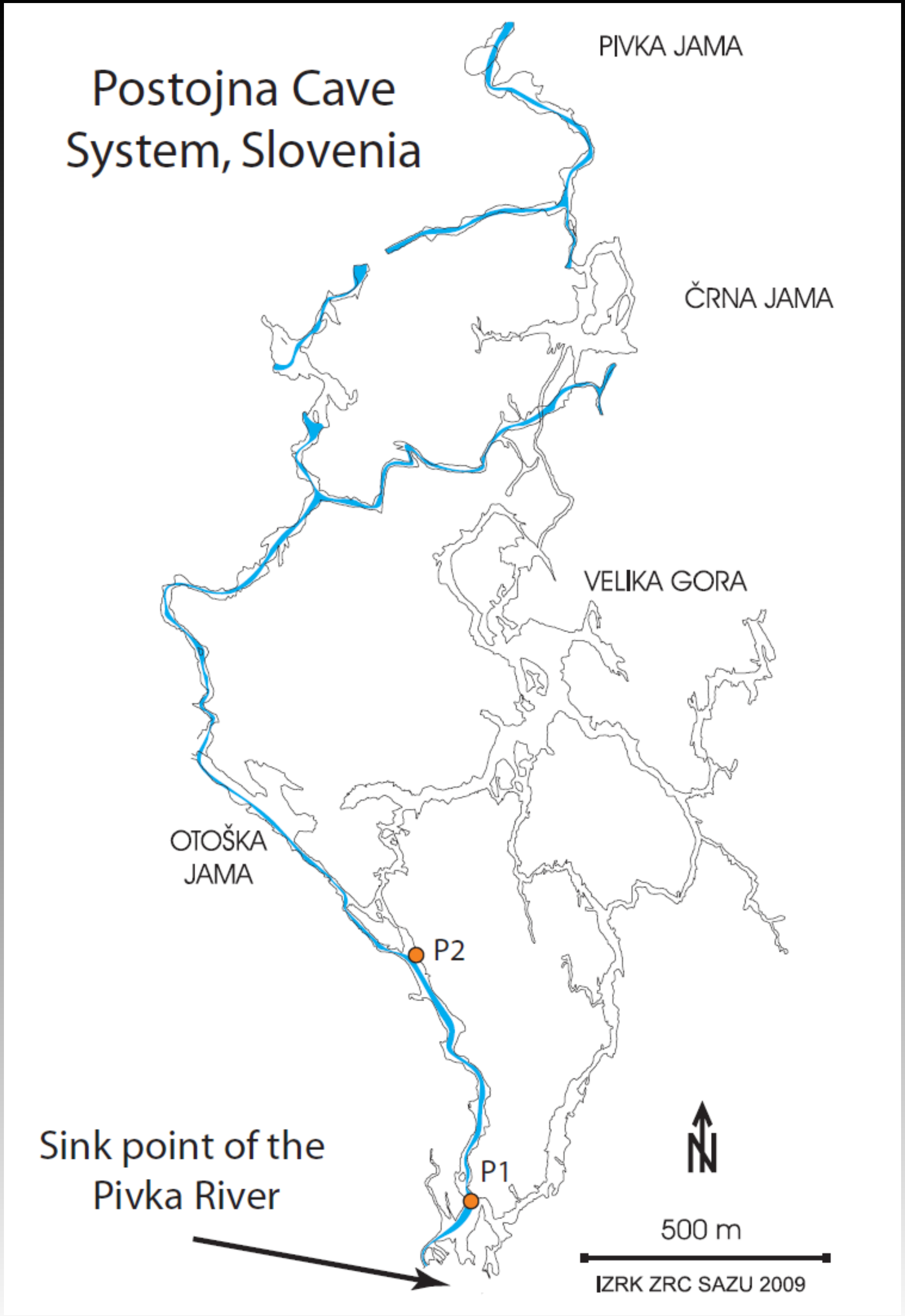
(Covington et al., in review, WRR)

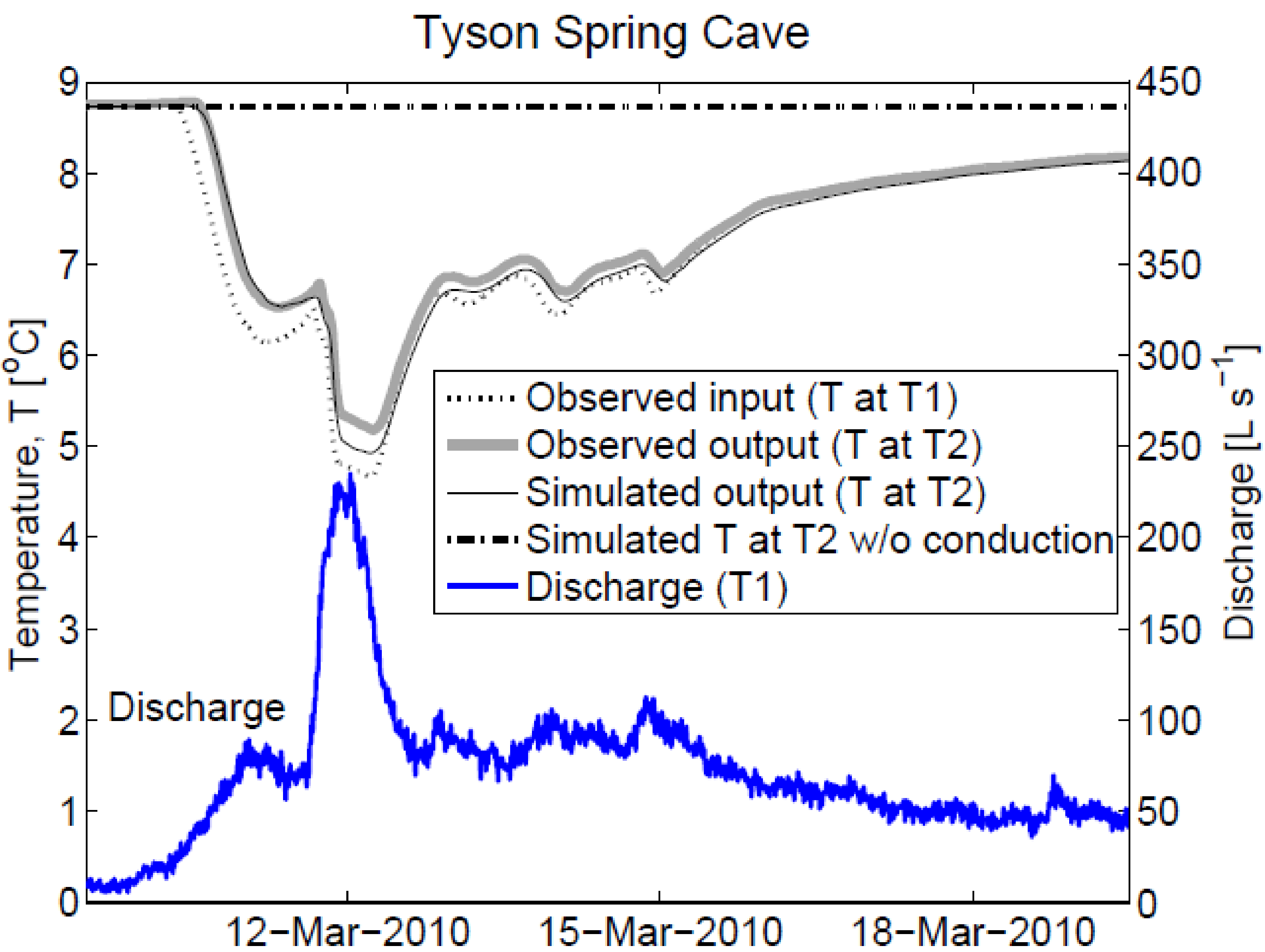

Tyson Spring Cave, MN

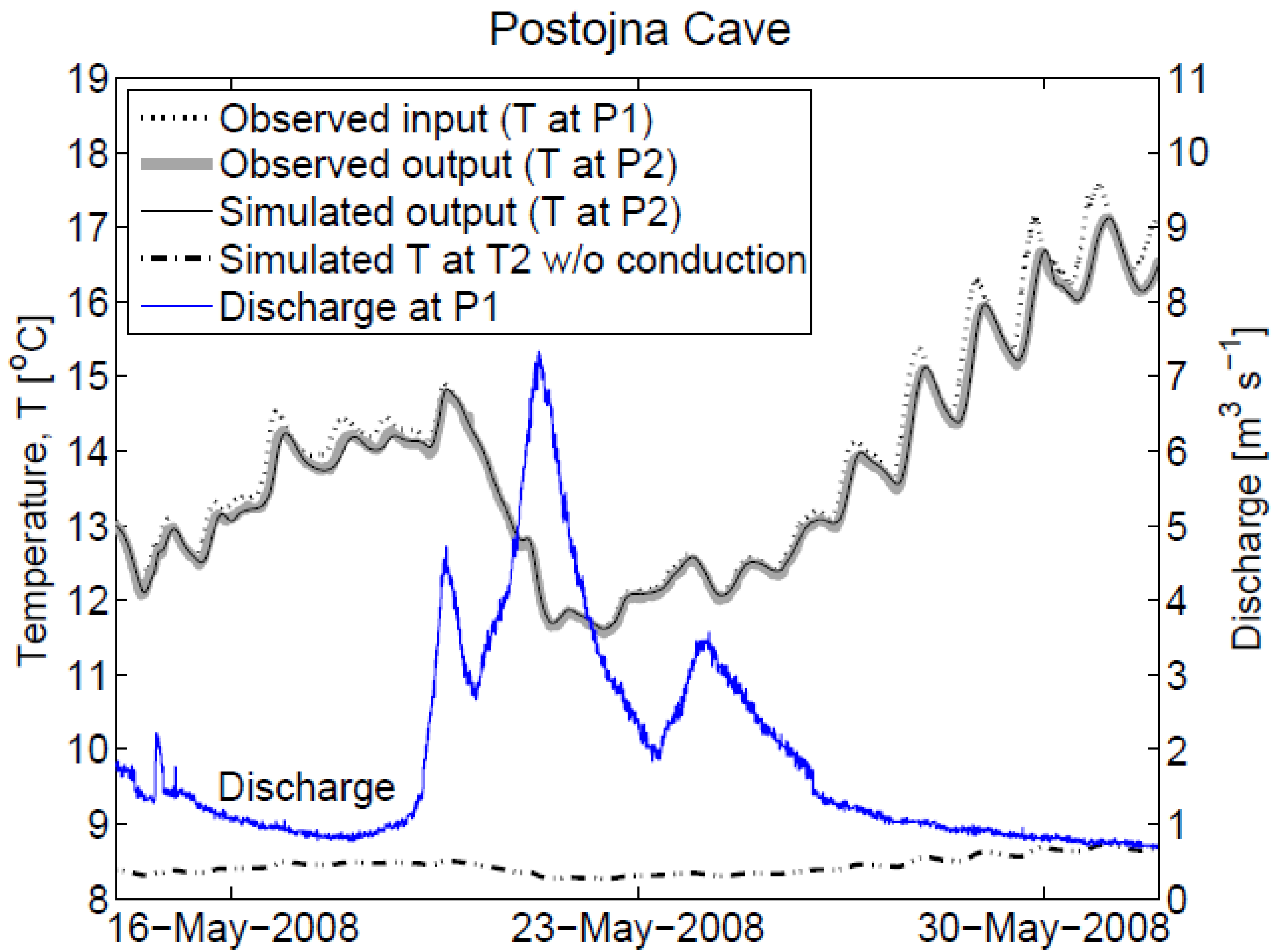


Autogenic recharge from a sinkhole plain

Allogenic recharge from a river




Tyson Spring Cave
Temperature, T [°C]
Discharge [L s^-1]
Observed input (T at T1)
Observed output (T at T2)
Simulated output (T at T2)
Simulated T at T2 w/o conduction
Discharge (T1)
Discharge
12−Mar−2010
15−Mar−2010
18−Mar−2010
0
1
2
3
4
5
6
7
8
9
0
50
100
150
200
250
300
350
400
450


Postojna Cave
Observed input (T at P1)
Observed output (T at P2)
Simulated output (T at P2)
Simulated T at T2 w/o conduction
Discharge at P1
Temperature, T [°C]
Discharge [m3 s-1]
Discharge
16−May−2008
23−May−2008
30−May−2008
19
18
17
16
15
14
13
12
11
10
9
8
11
10
9
8
7
6
5
4
3
2
1
0

How well do simulated and observed conduit diameters compare?

Postojna Cave:
  Simulated diameter = 1.5 m
  Average surveyed diameter = 2.0 m

Tyson Spring Cave
  Simulated diameter = 0.3 m
  Average surveyed diameter = 1.4 m

How well do simulated and observed conduit diameters compare?

Postojna Cave:
- Simulated diameter = 1.5 m
- Average surveyed diameter = 2.0 m

Tyson Spring Cave
- Simulated diameter = 0.3 m
- Average surveyed diameter = 1.4 m

Diffuse recharge??

Conclusions

- The relative importance of heat exchange mechanisms is a function of time. Rock conduction dominates the heat exchange at the water-rock boundary after only a few seconds.

- Conduction dominates the exchange at the rock-air boundary at times scales on the order of days to weeks.

- Conduction cannot be ignored in models of conduit heat exchange.

- Temperature provides a potential means of constraining diffuse recharge into conduits.