The Amazon Watershed - Sparse Regularization of Inverse Gravimetry

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Introduction

The Amazon watershed is one of the largest watersheds on Earth. Thus, it is very important to observe the mass transport in this area regularly.

The satellite mission GRACE was started in 2002 to gain more information about the Earth's gravitational potential which allows us to detect climate phenomena like hydrology in the gravity field. Furthermore, the GRACE mission provides us with a monthly coverage of the gravitational potential such that we are able to investigate temporal variations, too.

We apply a new regularizing method to reconstruct the mass transport in the Amazon watershed out of the gravitational potential given by satellite data. In our results, we observe the seasonal changes as well as a topographic and meteorological separation of some effects.

In general, the method, which is based on a matching pursuit and sparse regularization techniques, allows us to solve exponentially ill-posed problems stepwise. The resulting model has a resolution that is adapted to the data density as well as the detail density.

The Regularized Functional Matching Pursuit (RFMP)

The main idea is to choose dictionary functions $d_k \in \mathcal{D}$ and corresponding coefficients $\alpha_k \in \mathbb{R}$ stepwise such that they minimize

$$\sum_i (\mathcal{F}^i F_n - y_i)^2 + \lambda ||F_n||^2_{\mathrm{L}^2(\mathcal{B})}, \quad F_n = \sum_{k=1}^{n} \alpha_k d_k$$

where λ is the regularization parameter.

We construct an algorithm to solve an inverse problem on the ball:

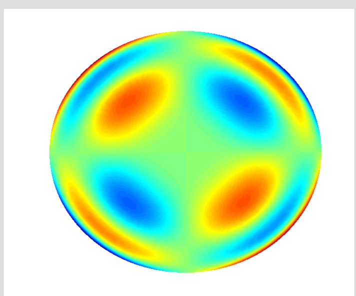
- Start with $F_0 = 0$.
- Build $F_{n+1} = F_n + \alpha_{n+1} d_{n+1}$ such that for the residual $R^n := \mathcal{F} F_n - y$

$$d_{n+1} \textit{ maximizes } \left| \frac{\langle R^n, \mathcal{F} d \rangle_{\mathbb{R}^l} - \lambda \langle F_n, d \rangle_{\mathrm{L}^2(\mathcal{B})}}{\sqrt{||\mathcal{F} d||^2_{\mathbb{R}^l} + \lambda ||d||^2_{\mathrm{L}^2(\mathcal{B})}}} \right| \textit{ and}$$

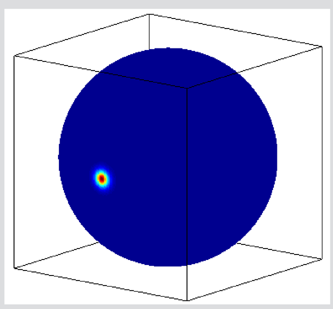
$$\alpha_{n+1} = \frac{\langle R^n, \mathcal{F} d_{n+1} \rangle_{\mathbb{R}^l} - \lambda \langle F_n, d_{n+1} \rangle_{\mathrm{L}^2(\mathcal{B})}}{||\mathcal{F} d_{n+1}||^2_{\mathbb{R}^l} + \lambda ||d_{n+1}||^2_{\mathrm{L}^2(\mathcal{B})}}.$$

As dictionary functions d we use:

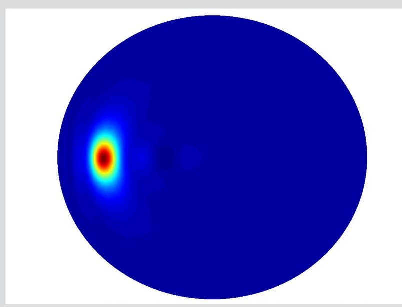
- the $\mathrm{L}^2(\mathcal{B})$-basis functions $\{G^{\mathrm{I}}_{m,n,j}\}_{m,n \in \mathbb{N}_0;\ j=1,\ldots,2n+1}$ to reconstruct global trends and
- the localized kernel functions $\{K_h(\cdot, x)\}_{h \in]0,1[}$ centred at x to reconstruct detail structures



$G^{\mathrm{I}}_{2,4,2}$ (plane cut)



$K_{0.9}(\cdot, x)$ (sphere)



$K_{0.8}(\cdot, x)$ (plane cut)

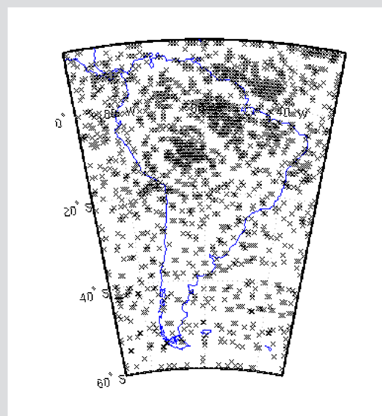
Numerical Application: Mass Transport in the Amazon Area

We use the following key data for our application:

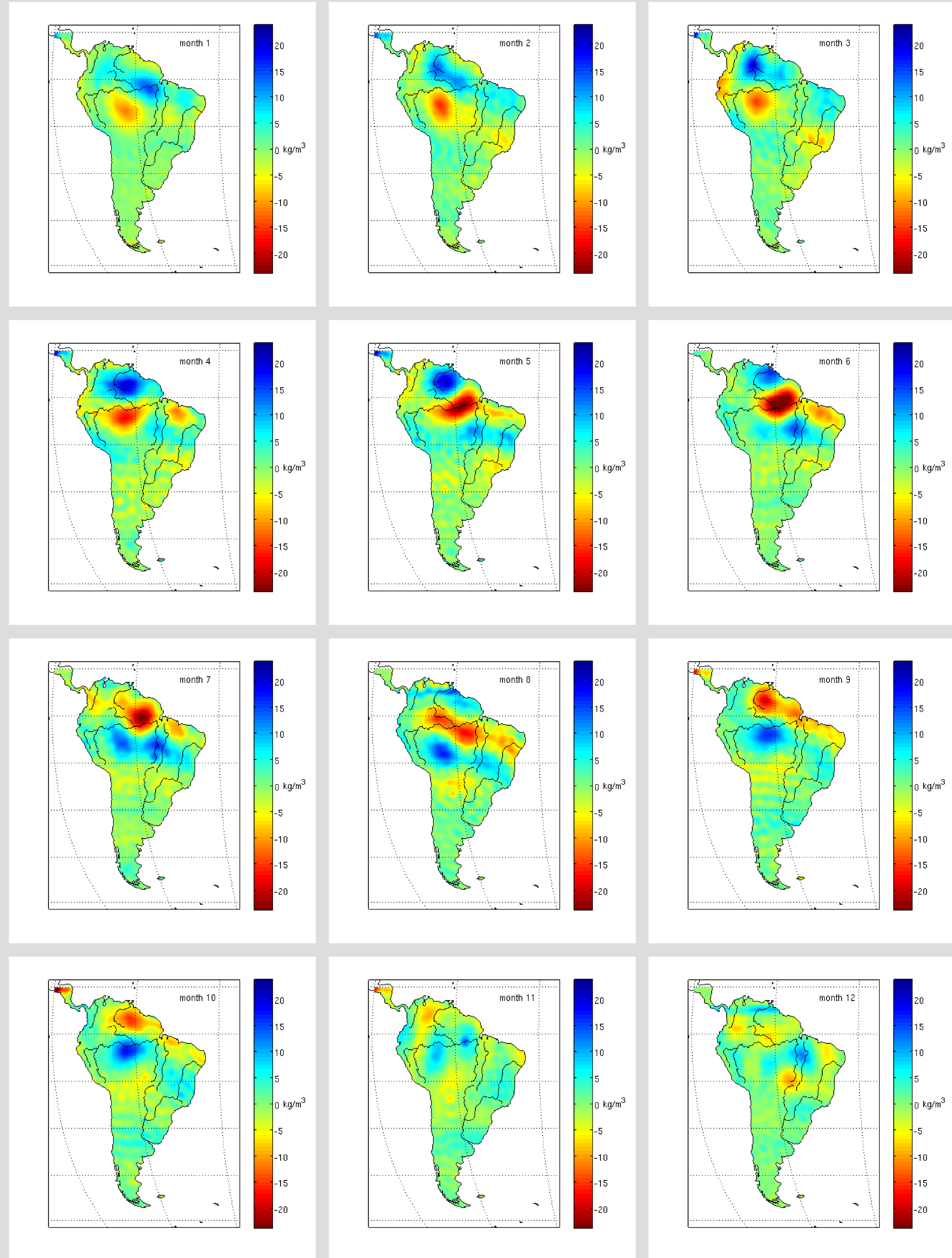
- reconstruction of the deviation of the monthly determined density distributions to the mean density distribution (2004 to 2009) for 2008
- inversion of potential values $y_i = V(x_i) - V_{\mathrm{mean}}(x_i)$ at 11,900 data points where we apply a CuP-filter for denoising
- regularization parameter $\lambda = 8.712$ chosen with an adapted L-curve method
- choice of 10,000 expansion functions out of the dictionary

On the right-hand side, we display the resulting density deviations for January 2008 (upper left-hand side) up to December 2008 (lower right-hand side). Here, the color blue denotes that the humidity is higher than in the mean, i.e. the surface and ground water levels are higher than in the mean.

Overall, the displayed results conform to empiric data from a temporal perspective as well as from a spatial one, i.e. the changes appear in accordance with the seasons in the Amazon area (compare e.g. April and September to detect seasonal changes) and the equator seems to be a natural interface for the change of conditions. Moreover, we observe a clear separation of the Amazon watershed and the Orinoco watershed in the north of South America which is a very important feature to be reconstructed, since we do not only have a meteorological separation by the equator but also a topographic separation by the Guiana highlands.



Here we display the centres x of the chosen localized kernel functions $K_h(\cdot, x)$ for January 2008. Clearly, the expansion functions are primarily chosen in accordance with the detail density of the solution which is one of the main features of the new algorithm.



upper left-hand: January 2008, upper right-hand: March 2008

Summary

The new method has certain advantages over already existing ones:

- we have an adaptive and iterative method to reconstruct, for example, the density distribution of the Earth
- the solution is matched to the structure of the target function
- with a step-by-step improvement we can refine the solution or zoom in on places of interest
- we can directly control the sparsity of the solution
- parts of the algorithm can be parallelized
- we can combine different types of expansion functions and different types of data

Outlook

Of course, the method may be applied to other ecologically relevant problems as well, e.g. the deglaciation of Antarctica and Greenland.

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