

INTERACTING NETWORKS - THEORY AND APPLICATION TO COUPLED CLIMATE NETWORKS

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SUMMARY

Interacting networks or networks of networks are frequently found in nature, technology and society. We introduce a novel framework for the statistical analysis of their topology [1]. This allows to investigate the interdependency structure between different fields of climatological variables using coupled climate network analysis [1]. Here we use this approach to study the Earth's atmosphere's vertical dynamical structure and uncover features of its stratification and general circulation.

INTERACTING NETWORKS

Interacting network measures allow to quantify the structural role of **single vertices** or **whole subnetworks** with respect to the interaction of a pair of non-overlapping subnetworks $G_i = (V_i, E_i)$, $G_j = (V_j, E_j)$ (Fig. 1). E_{ij} gives their interaction structure. The full network $G = (V, E)$ is described by the adjacency matrix A_{pq} .

Cross-degree centrality gives the number of direct neighbours of vertex $v \in V_i$ in G_j

$$k_v^j = \sum_{q \in V_j} A_{vq}$$

Cross-edge density measures the degree of connectivity of two subnetworks

$$\rho_{ij} = \rho_{ji} = \frac{\langle k_v^j \rangle_{v \in V_i}}{|V_j|}$$

The **global cross-clustering coefficient** is an estimate of the probability of vertices from G_i to have mutually connected neighbours within G_j

$$C_{ij} = \frac{1}{|V_i|} \sum_{\substack{v \in V_i, \\ k_v^j > 1}} \frac{\sum_{p \neq q \in V_j} A_{vp} A_{pq} A_{qv}}{\sum_{p \neq q \in V_j} A_{vp} A_{vq}}$$

For any vertex w in V , **cross-betweenness** indicates its role for mediating communication between G_i and G_j along unconstrained shortest paths.

REFERENCES

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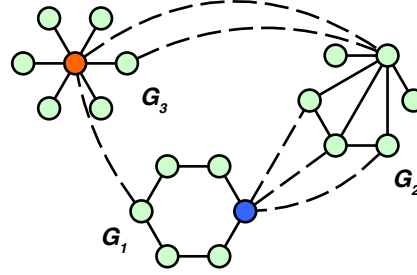


Fig. 1 A network of interacting networks. Blue vertex has high cross-degree $k_v^{1,2}$, red vertex shows increased cross-betweenness $b_w^{1,2}$.

COUPLED CLIMATE NETWORKS

Coupled climate networks map the statistical associations within and between different climatological fields. Vertices represent measurement stations or grid points where data is available in form of time series. Two vertices are linked by an edge if a significant statistical relationship is detected between the respective time series [1-3].

Here we study the correlation structure of the vertically resolved geopotential height field $Z(t)$ from NCEP/NCAR Reanalysis 1 data. We identify subnetworks with quasi-horizontal isobaric surfaces. Our interest is in the interaction structure between the near-ground level and higher isobaric surfaces (1000 to 10 mb) (Fig. 2).

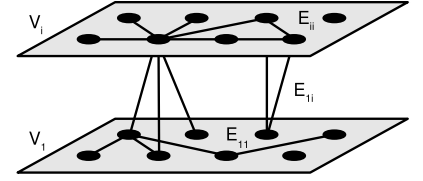


Fig. 2 Illustration of a coupled climate network.

RESULTS

Cross-edge density and global cross-clustering show aspects of the atmosphere's stratification (e.g., planetary boundary layer, tropopause) (Fig. 3).

Zonally averaged cross-degree and cross-betweenness reveal main features of the atmosphere's general circulation (Hadley cells, polar vortex) (Fig. 4).

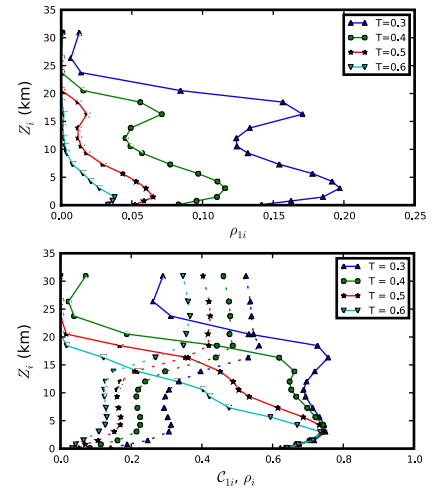


Fig. 3 Cross-edge density and "upward" global cross-clustering for geopotential height coupled climate networks.

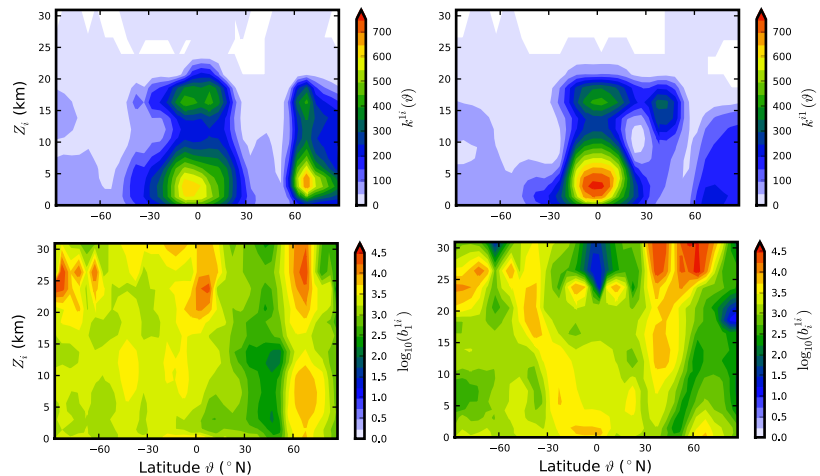


Fig. 4 Zonally averaged cross-degree centrality (pointing upward and downward) and cross-betweenness (near-ground and upper component). Here an edge inclusion threshold of 0.4 Pearson correlation was used.

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