



# **Joint inversion of gravity data together with Pg traveltimes**

from shots and Pg/Sg onsets from earthquakes in the  
Western Bohemia/Vogtland swarm region

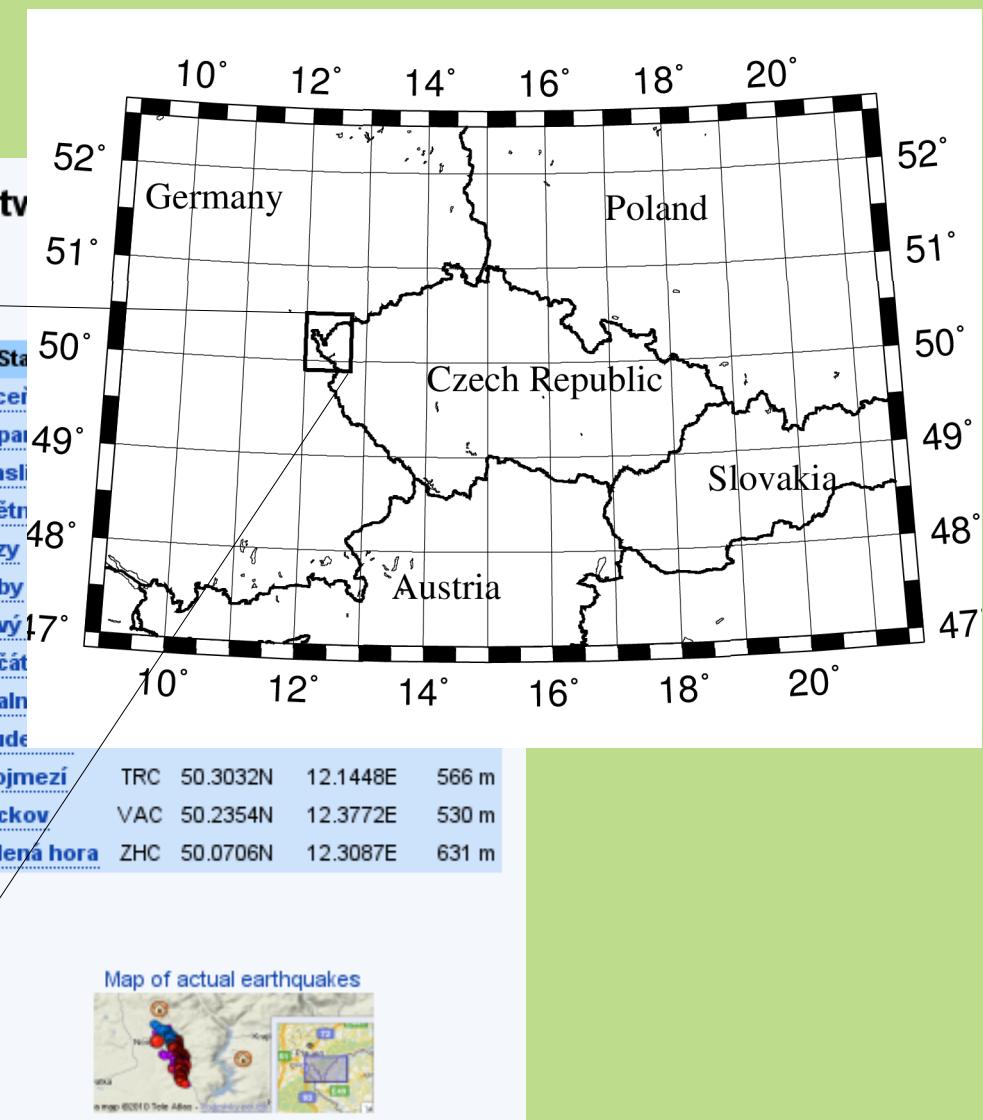
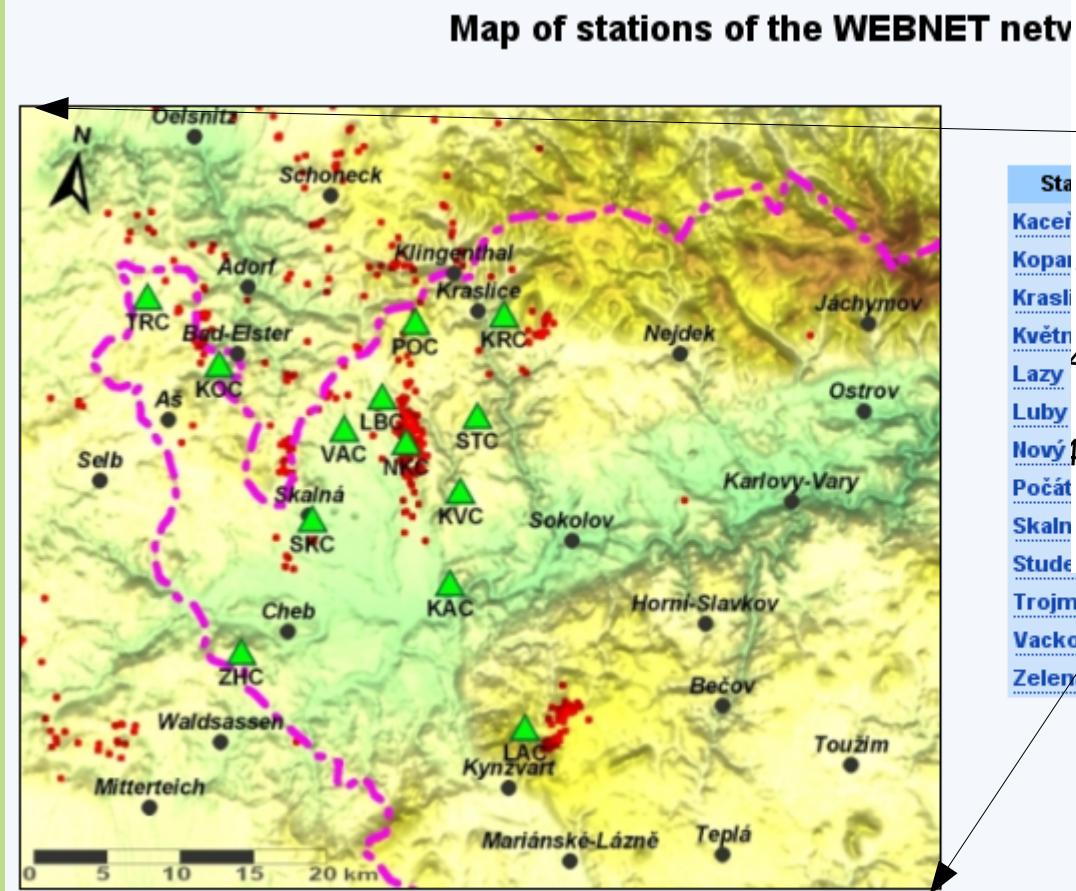
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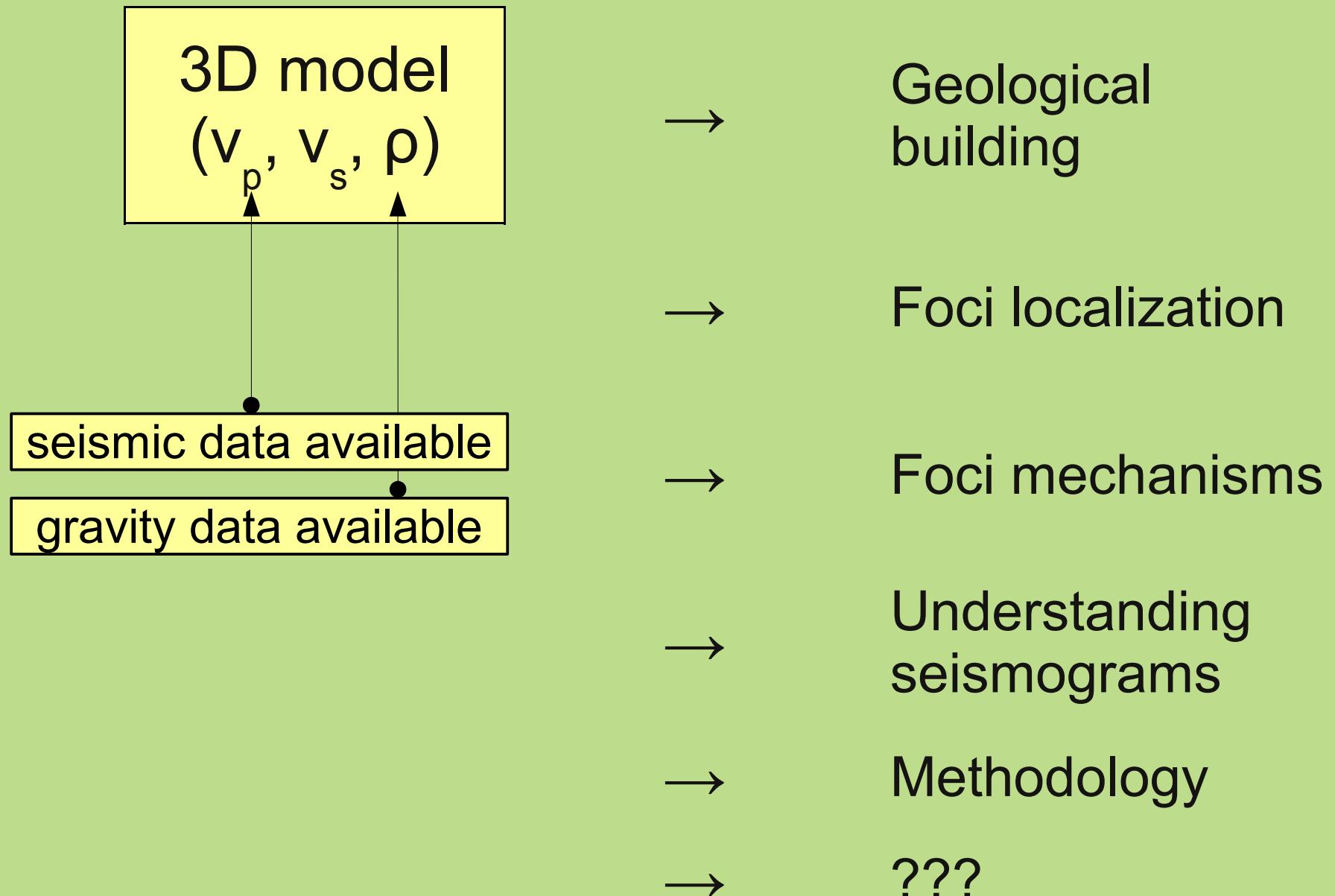
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EGU General Assembly, Vienna, 04-08 April 2011

Western Bohemia/Vogtland region is known for the occurrence of seismic swarms,

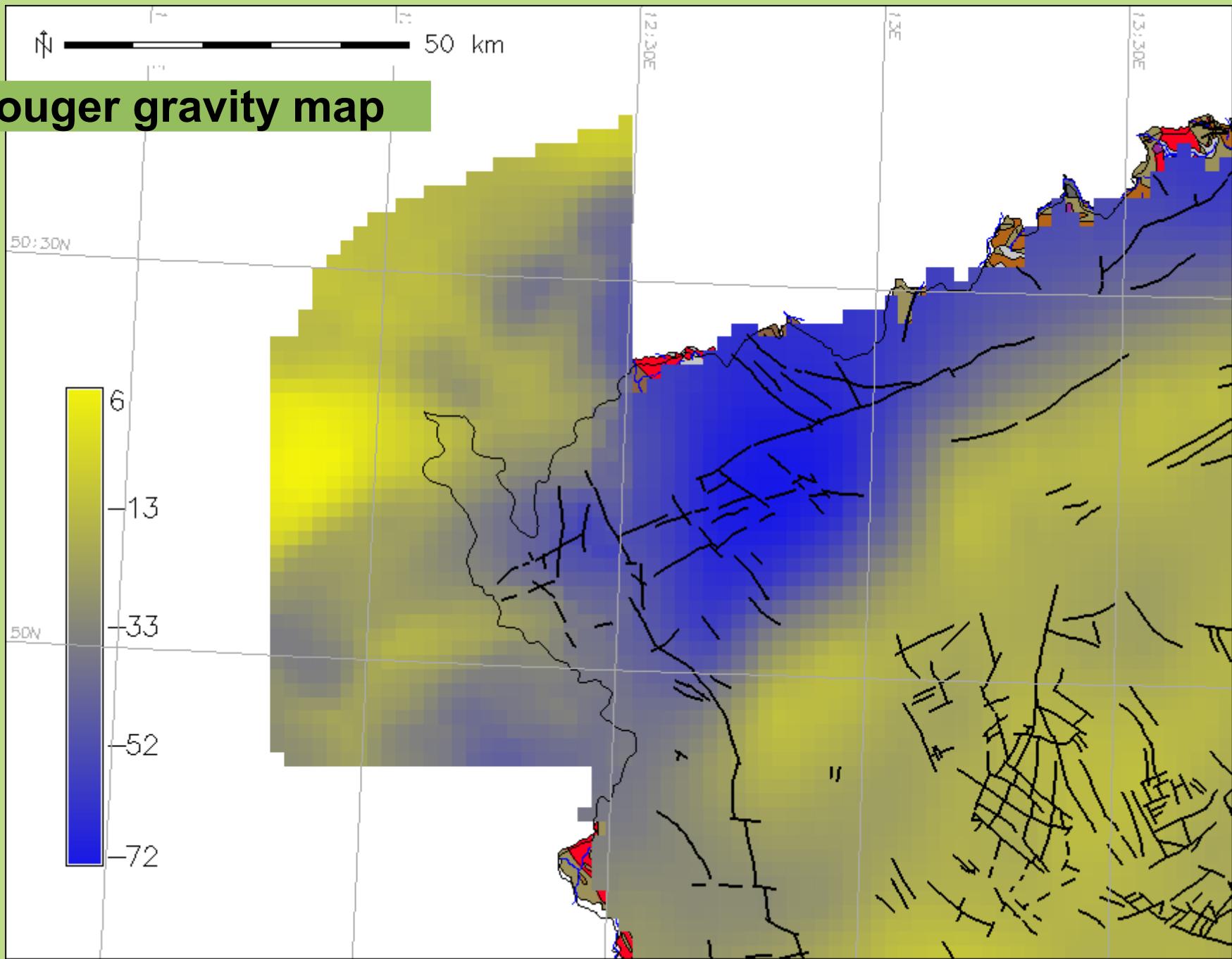


but till now no commonly accepted 3D model is available ...



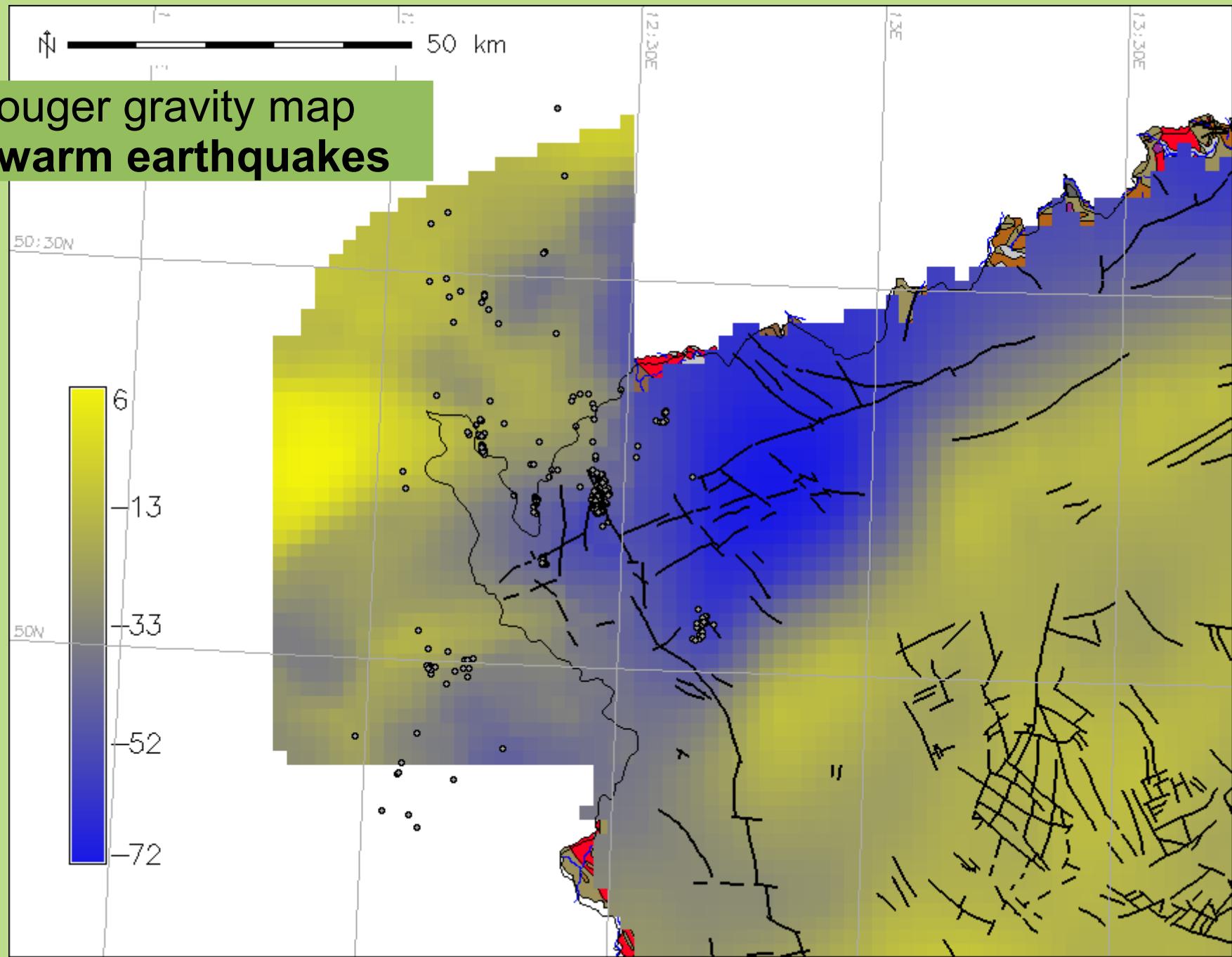
# Contents

- Data sets available
- Methodology
- Test
- Indicative result

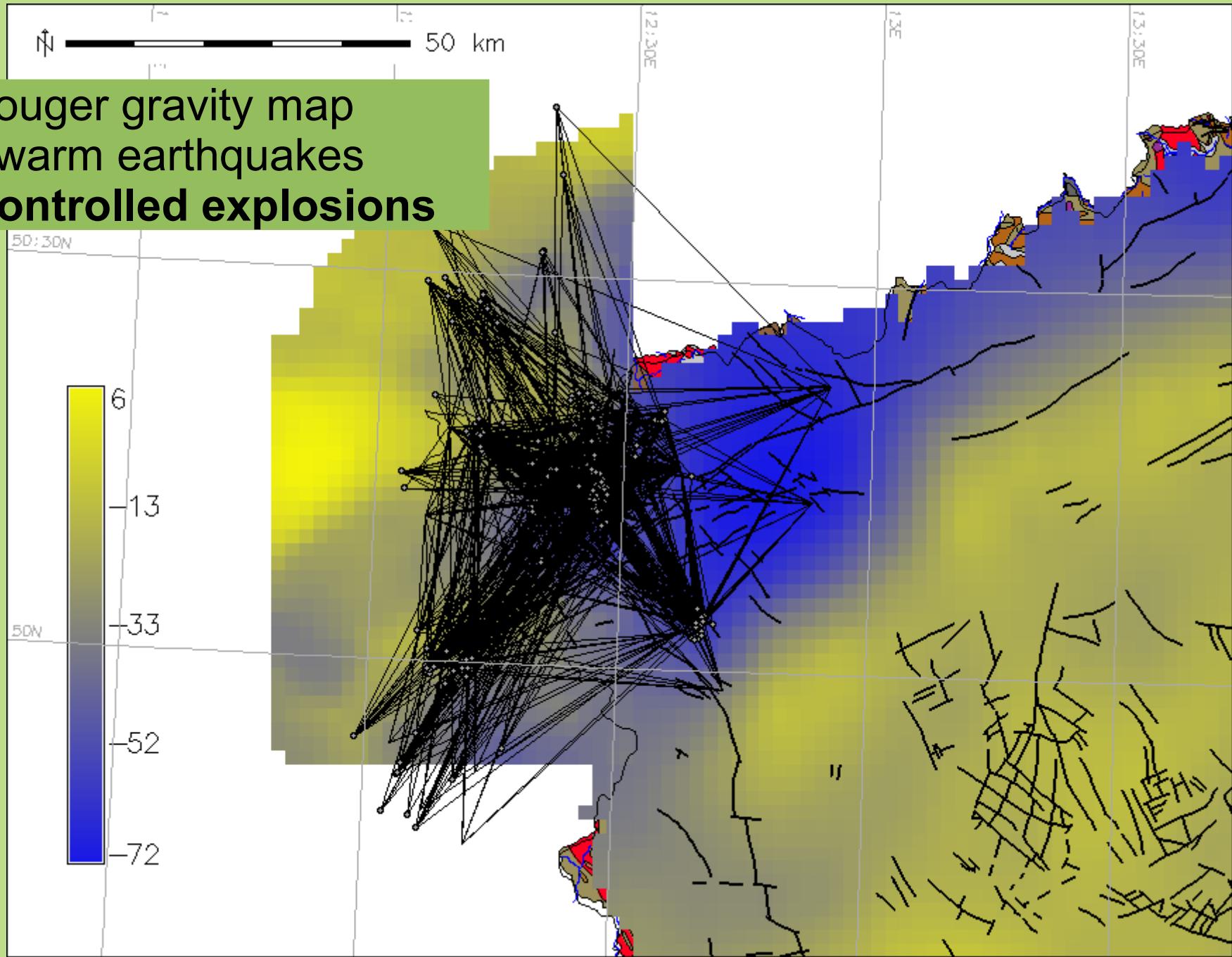


- **Bouguer gravity map**

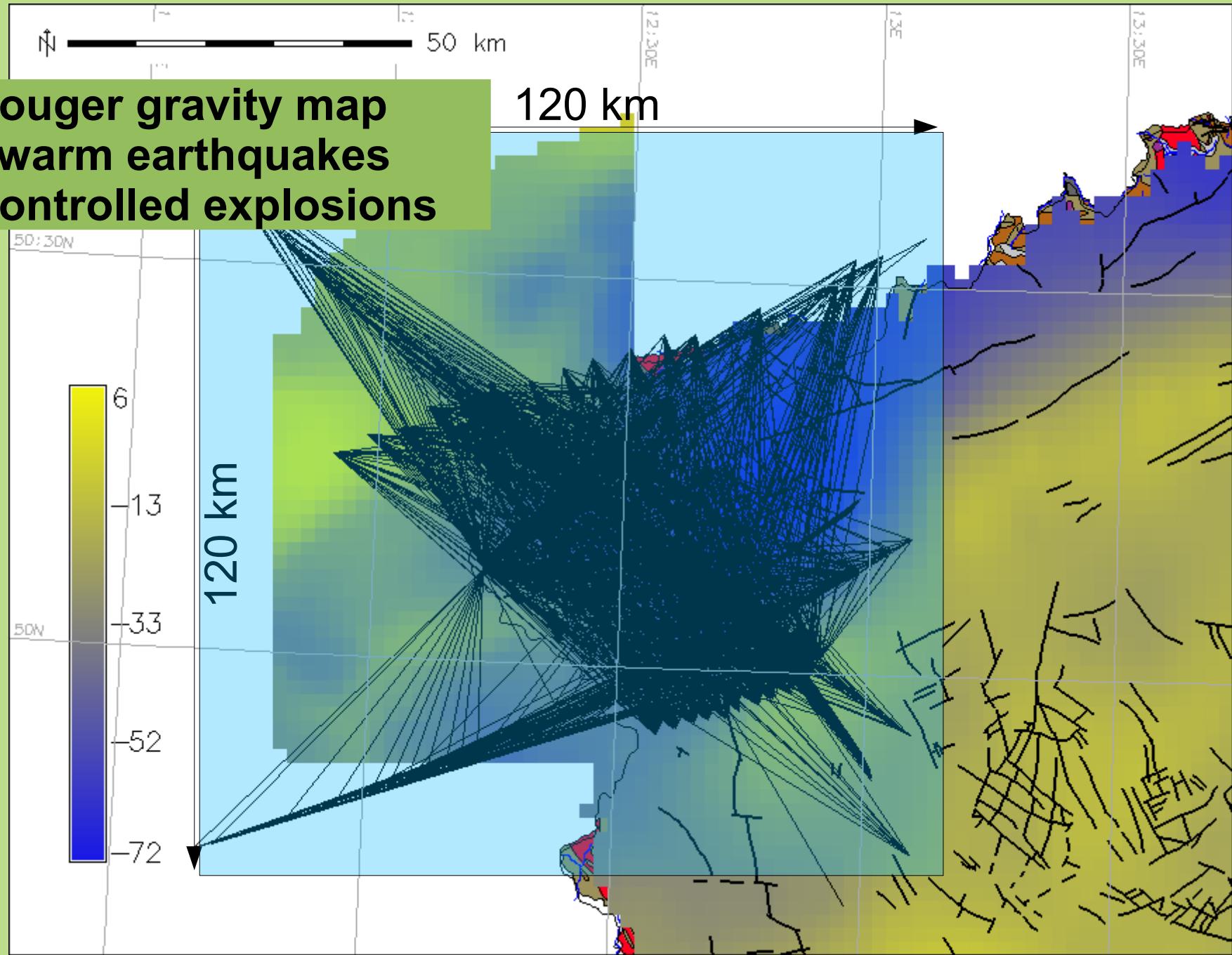
- Bouger gravity map
- Swarm earthquakes



- Bouger gravity map
- Swarm earthquakes
- **Controlled explosions**

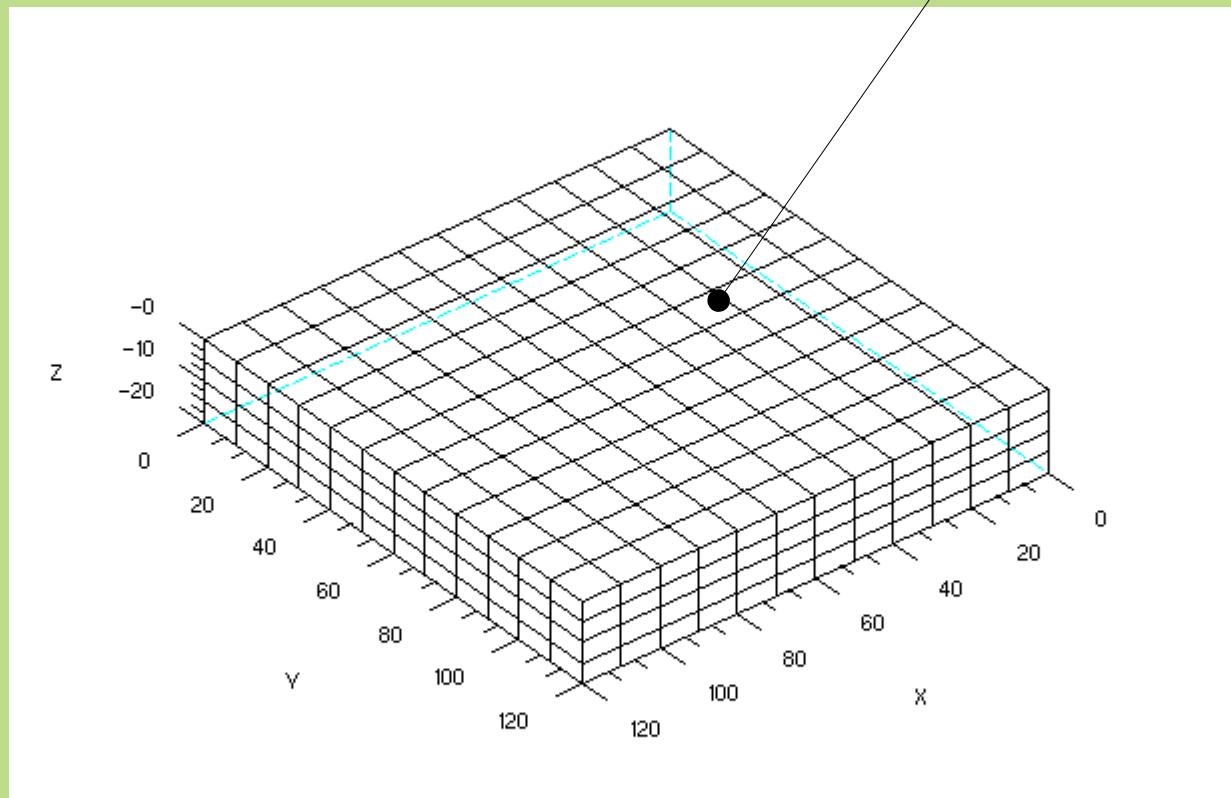


- Bouger gravity map
- Swarm earthquakes
- Controlled explosions



## Globe $\leftrightarrow$ rectangle Lambert projection

$(v_p, v_s, \rho)$



$dx = dy = dz = 2 \text{ km}$   
 $nx = 48$   
 $ny = 54$   
 $nz = 17$   
**44064 cells**

## Data space

2821 rays from explosions (P)

6405 rays from earthquakes (P)

6716 rays from earthquakes (S)

1371 gravity measurements

n = 17323

## Model space

732 earthquake localizations (x,y,z,T)

44064 x ( $v_p$ ,  $v_s$ ,  $\rho$ )

$m = 732 \cdot 4 + 54000 \cdot 3 = 135120$

$m \leq 135120$

Single inversion



$$\mathbf{g}(\mathbf{m}) = \mathbf{d}$$

$$\mathbf{d}^{obs}$$

$$\mathbf{m}^{sol}: \mathbf{g}(\mathbf{m}) \rightarrow \mathbf{d}^{obs}$$

Joint inversion of 1<sup>st</sup> kind



$$\mathbf{g}_1(\mathbf{m}) = \mathbf{d}_1$$

$$\mathbf{g}_2(\mathbf{m}) = \mathbf{d}_2$$

$$\begin{bmatrix} \mathbf{g}_1(\mathbf{m}) \\ \mathbf{g}_2(\mathbf{m}) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}$$

$$\mathbf{m}^{sol}: \begin{bmatrix} \mathbf{g}_1(\mathbf{m}) \\ \mathbf{g}_2(\mathbf{m}) \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{d}_1^{obs} \\ \mathbf{d}_2^{obs} \end{bmatrix}$$

Joint inversion of 2<sup>nd</sup> kind



$$\mathbf{g}_1(\mathbf{m}_1) = \mathbf{d}_1$$

$$\mathbf{g}_2(\mathbf{m}_2) = \mathbf{d}_2$$

$$\begin{bmatrix} \mathbf{g}_1(\mathbf{m}_1) \\ \mathbf{g}_2(\mathbf{m}_2) \\ \mathbf{g}_r(\mathbf{m}_1, \mathbf{m}_2) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{0} \end{bmatrix}$$

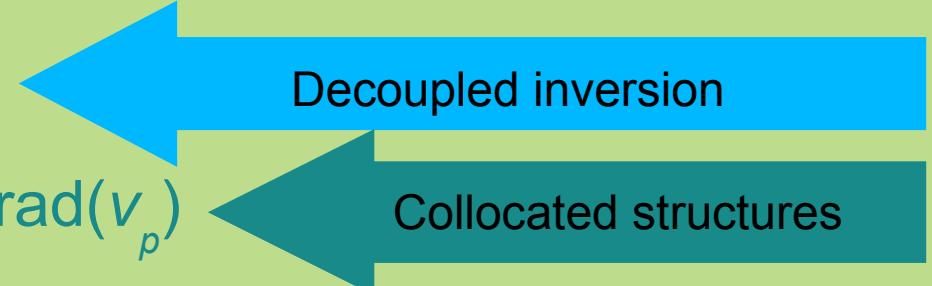
$$g_r(\mathbf{m}_1, \mathbf{m}_2): \rho = k_1 * v_p + k_2 \text{ etc.}$$

## “Cross-gradient method”

$$\mathbf{m}_1 \rightarrow \rho(x, y, z)$$

$$\mathbf{m}_2 \rightarrow v_p(x, y, z)$$

$$\nabla(\rho) * \nabla(v_p) = \mathbf{0}$$

- $\rho = \text{const}$
  - $v_p = \text{const}$
  - $\text{grad}(\rho) \parallel \text{grad}(v_p)$
- 

Gallardo, L. A., and M. A. Meju, 2004. Joint two-dimensional DC resistivity and seismic travel time inversion with cross-gradients constraints: *Journal of Geophysical Research*, **109**, B03311, doi: 10.1029/2003JB002716.

Tryggvason, A., and N. Linde, 2006. Local earthquake (LE) tomography with joint inversion for P- and S-wave velocities using structural constraints: *Geophysical Research Letters*, **33**, L07303, doi: 10.1029/2005GL025485.

Fregoso E. and Gallardo L.A., 2009. Cross-gradients joint 3D inversion with applications to gravity and magnetic data. *Geophysics* **74**, No. 4, P. L31–L42, 10.1190/1.3119263.

## Synthetic example

10 x 10 = 100 cells

90 P-measurements

90 S-measurements

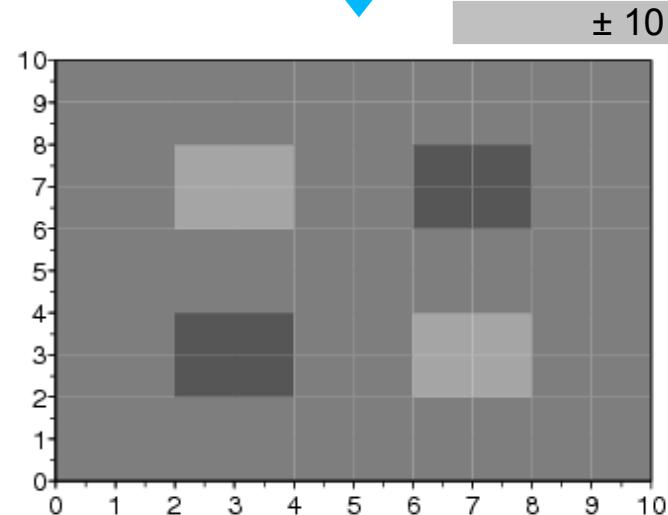
No constraints

Regularization via SVD

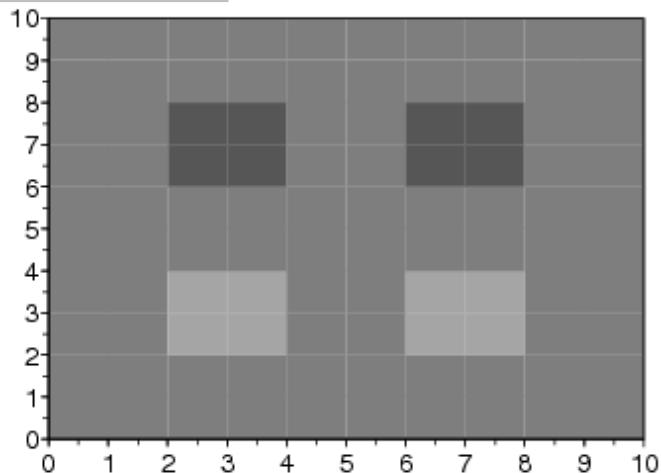
P-slowness

S-slowness

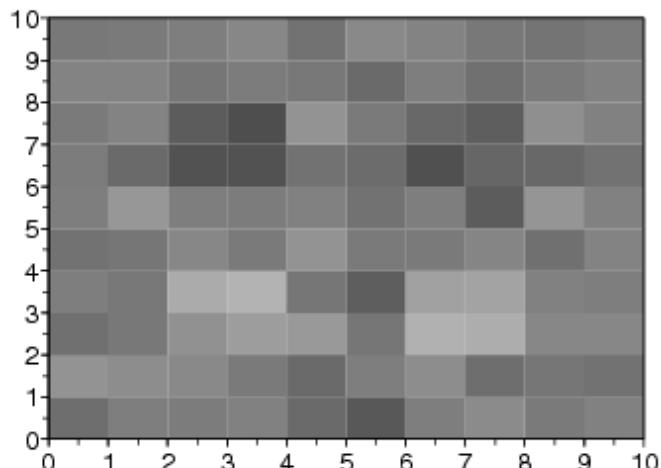
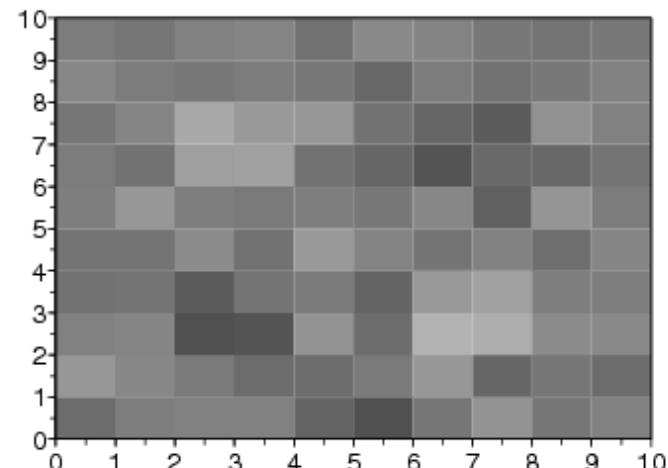
Input patterns



$\pm 10\%$  anomalies



Reconstruction



## Synthetic example

10 x 10 = 100 cells

90 P-measurements

90 S-measurements

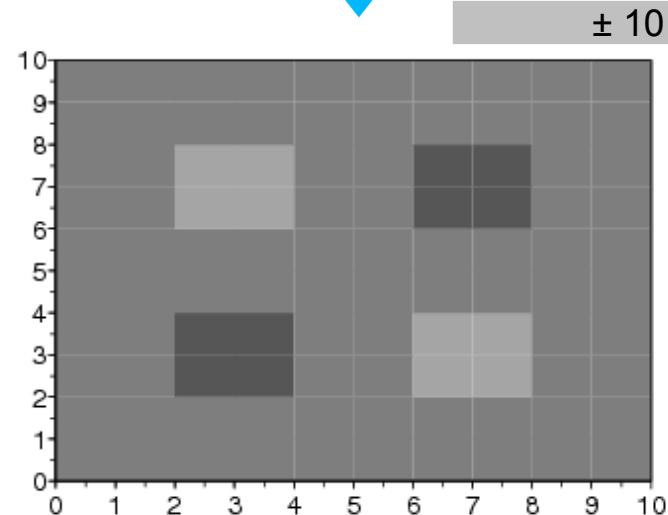
Cross-gradient constraints

Regularization via SVD

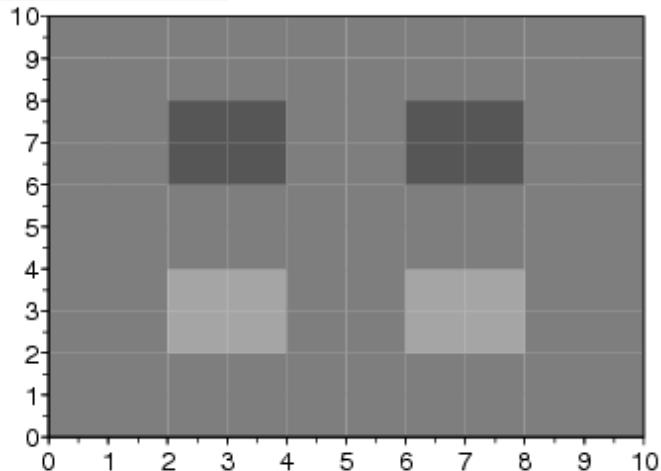
P-slowness

S-slowness

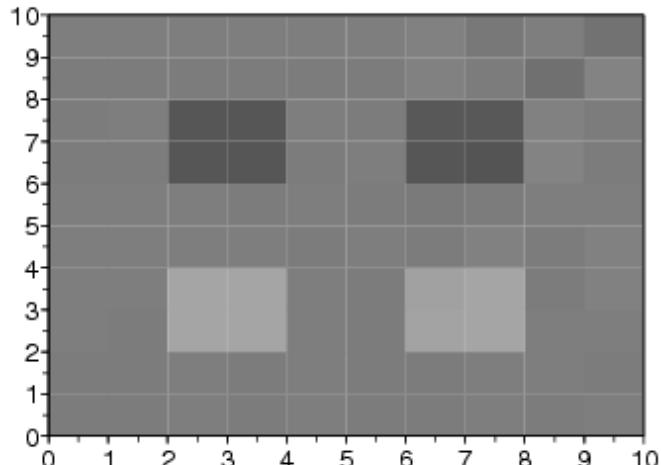
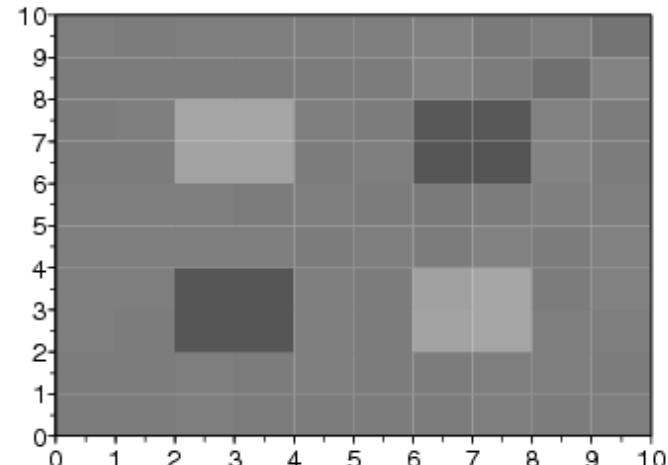
Input patterns



$\pm 10\%$  anomalies



Reconstruction



## Synthetic example

$$10 \times 10 = 100 \text{ cells}$$

## 90 P-measurements

## 90 S-measurements

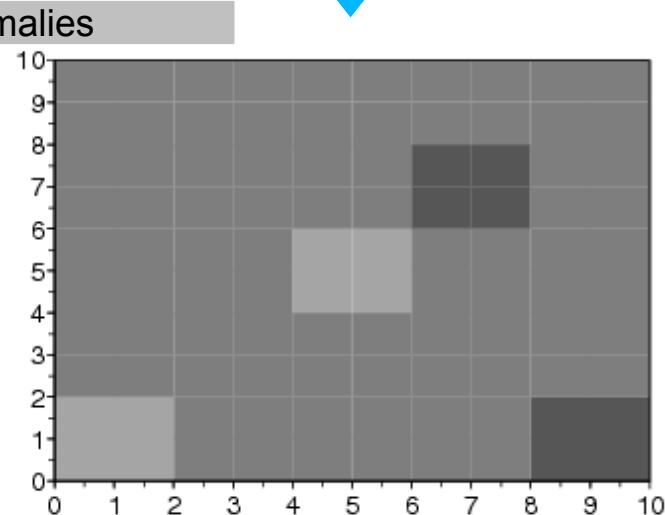
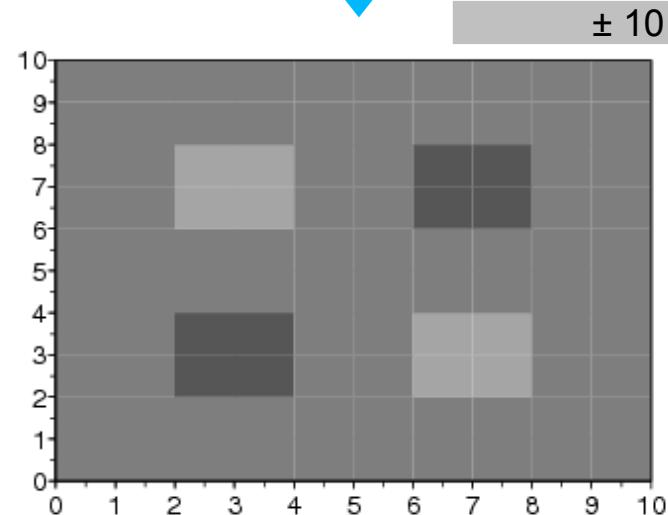
## Cross-gradient constraints

## Regularization via SVD

## P-slowness

## S-slowness

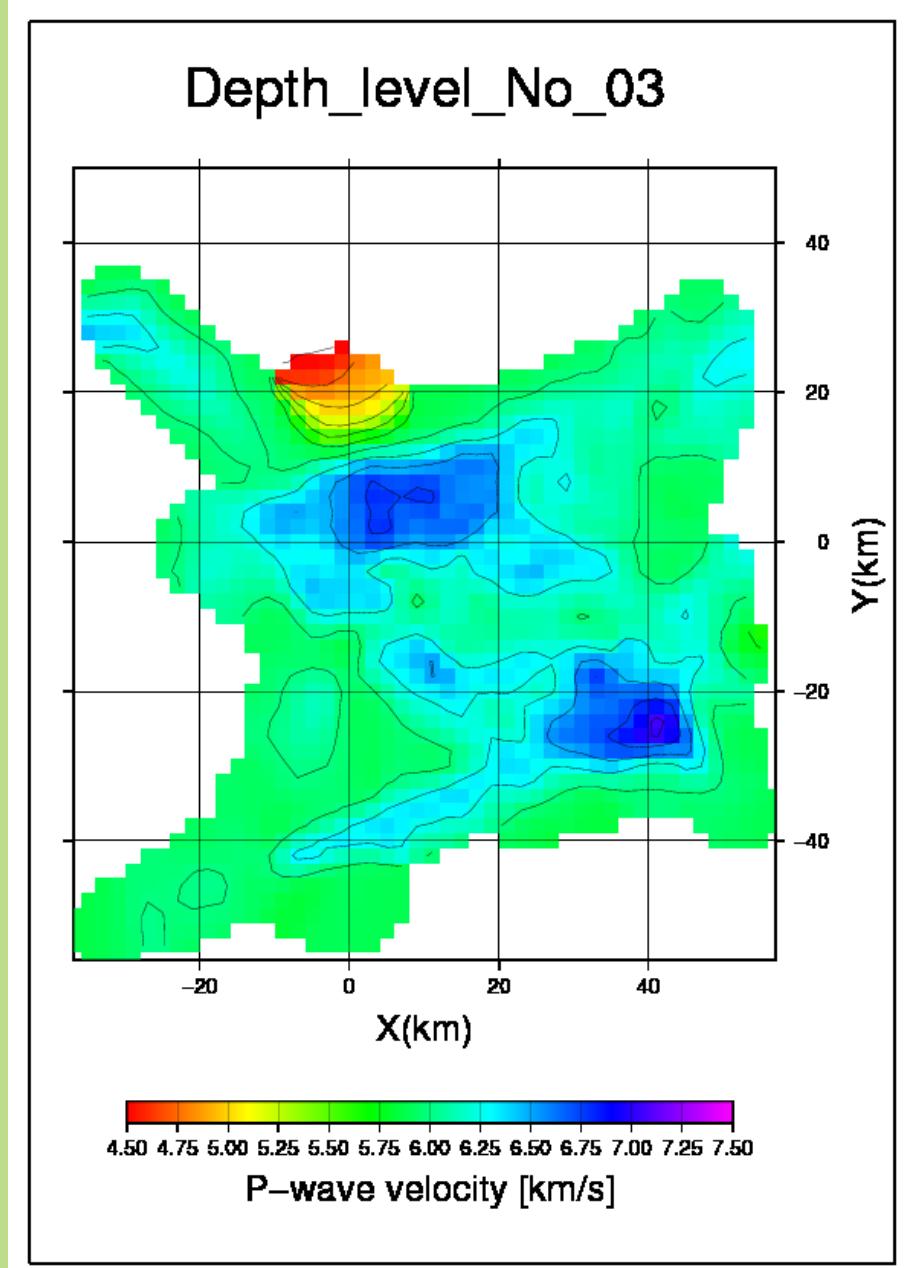
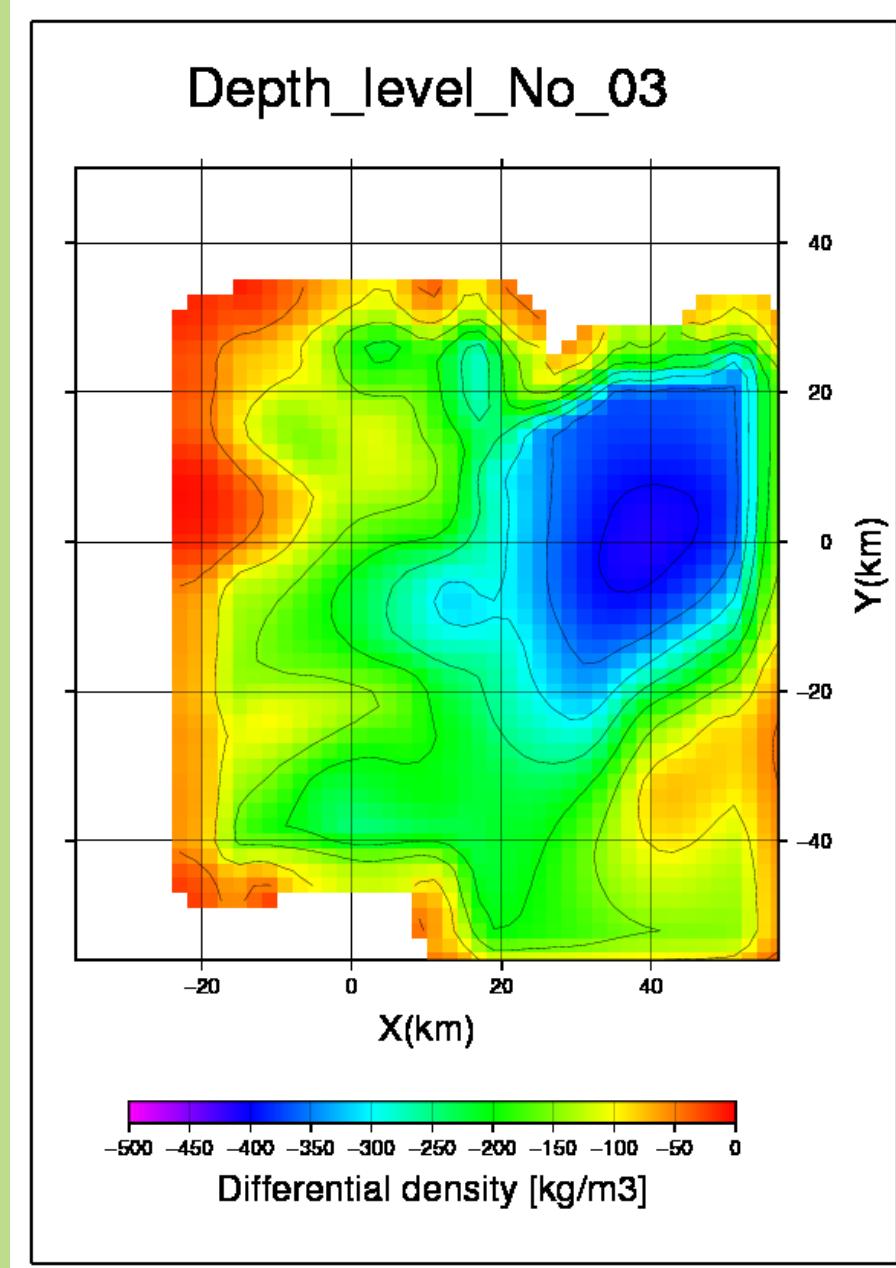
# Input patterns



# Reconstruction

Including cross-gradient constraint helps to discover structurally similar objects

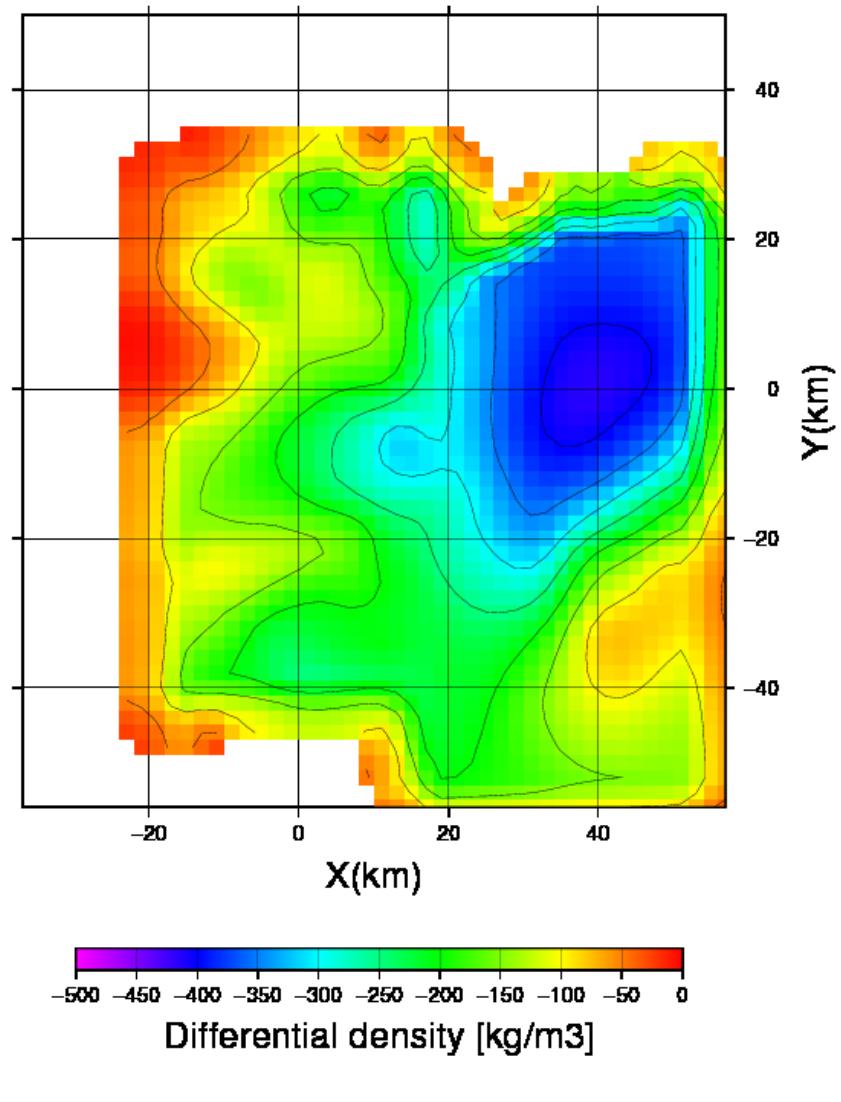
# Horizontal cross-sections in a depth of 4 km



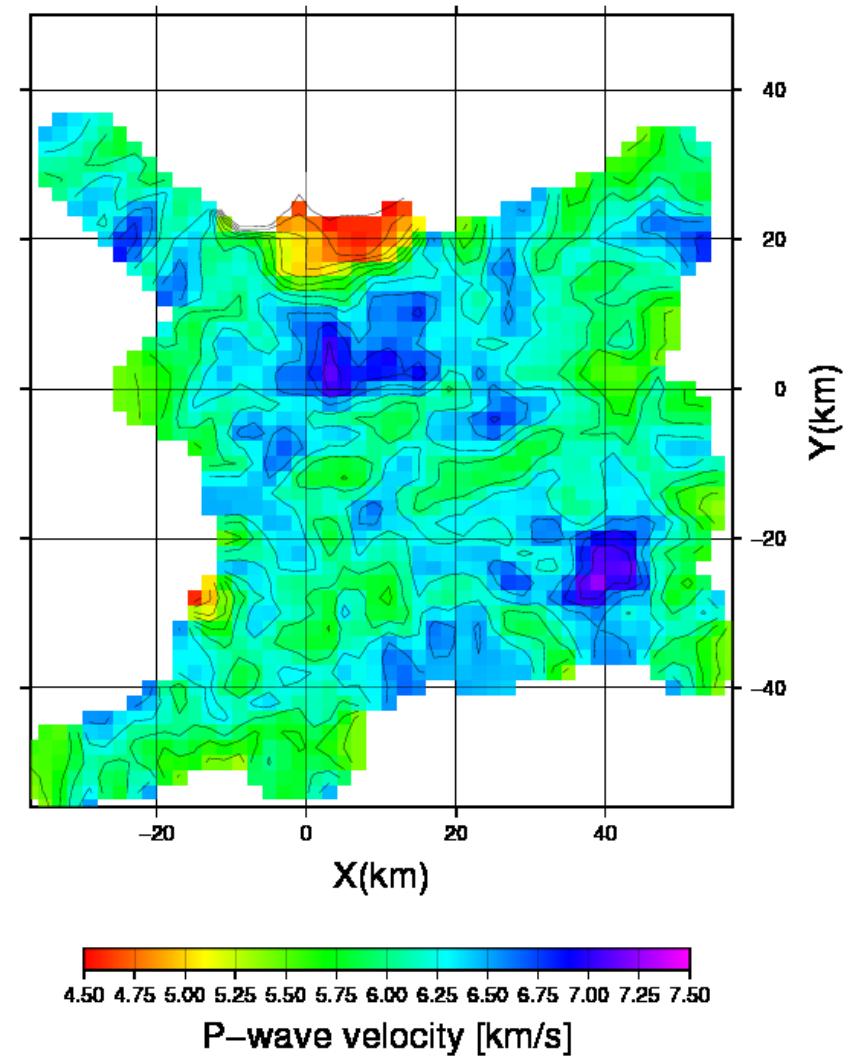
# Horizontal cross-sections in a depth of 4 km

$$\begin{bmatrix} g_1(m_1) \\ g_2(m_2) \\ g_r(m_1, m_2) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} \leftarrow \begin{bmatrix} \Delta t \approx 0.1 \text{ s} \\ \Delta g \approx 0.4 \text{ mGal} \\ \lambda \approx 1 * \text{norm} \end{bmatrix}$$

Depth\_level\_No\_03



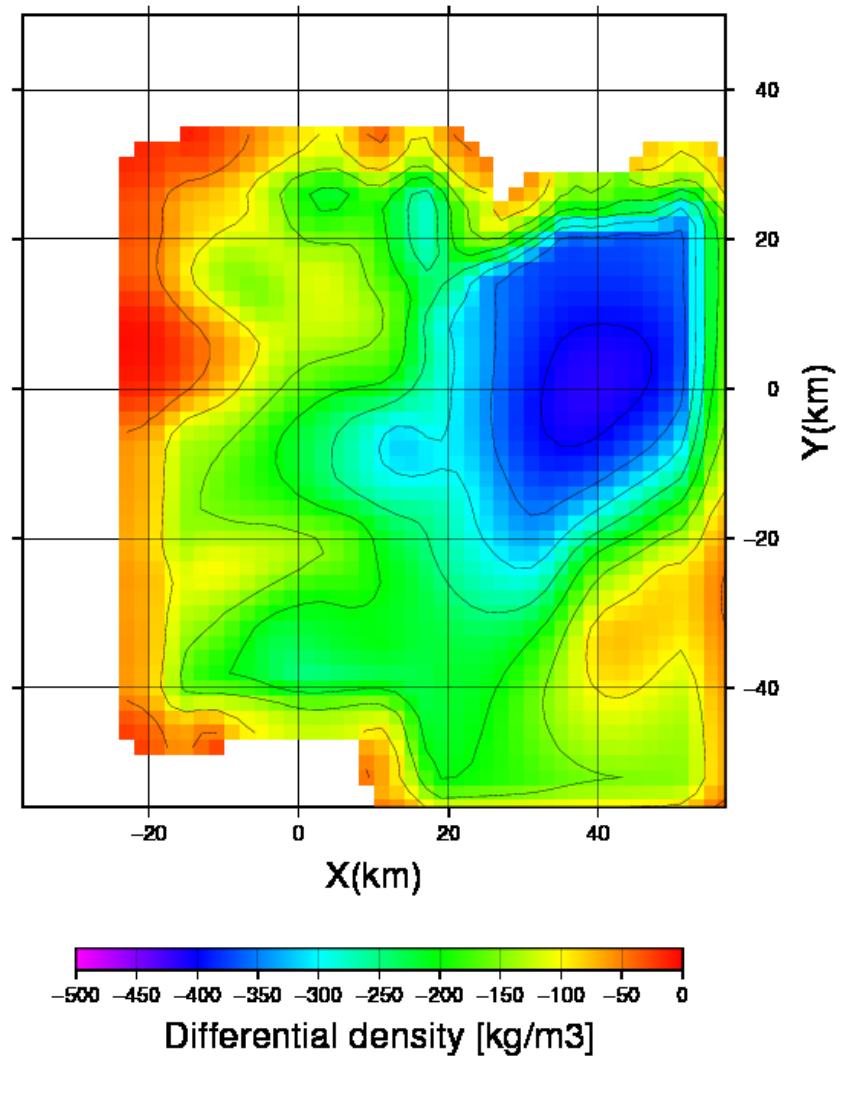
Depth\_level\_No\_03



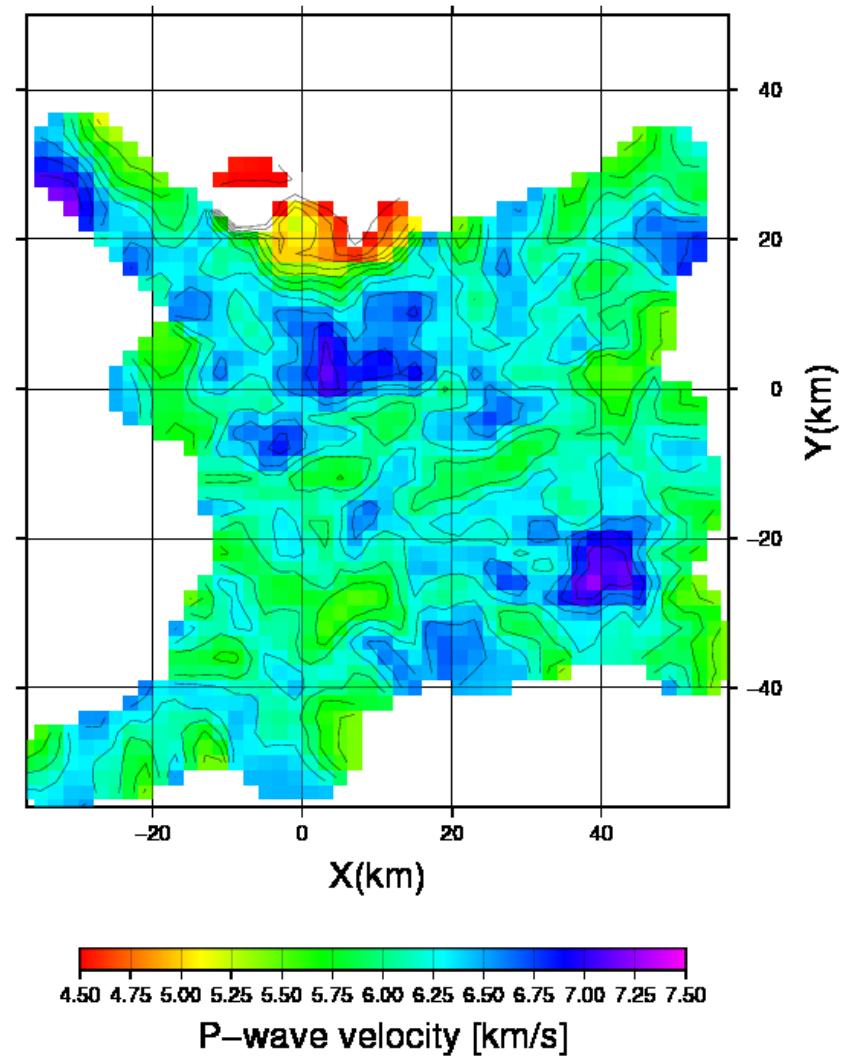
# Horizontal cross-sections in a depth of 4 km

$$\begin{bmatrix} g_1(m_1) \\ g_2(m_2) \\ g_r(m_1, m_2) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} \leftarrow \begin{bmatrix} \Delta t \approx 0.1 \text{ s} \\ \Delta g \approx 0.4 \text{ mGal} \\ \lambda \approx 2 * \text{norm} \end{bmatrix}$$

Depth\_level\_No\_03



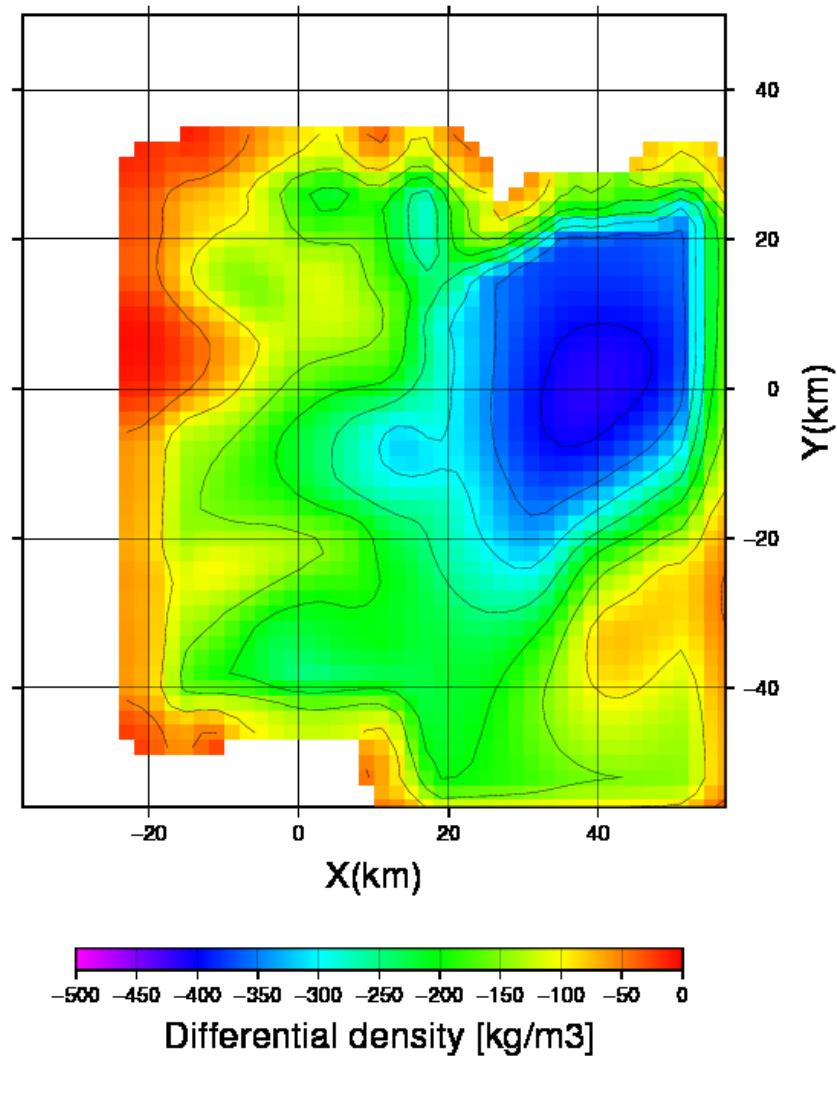
Depth\_level\_No\_03



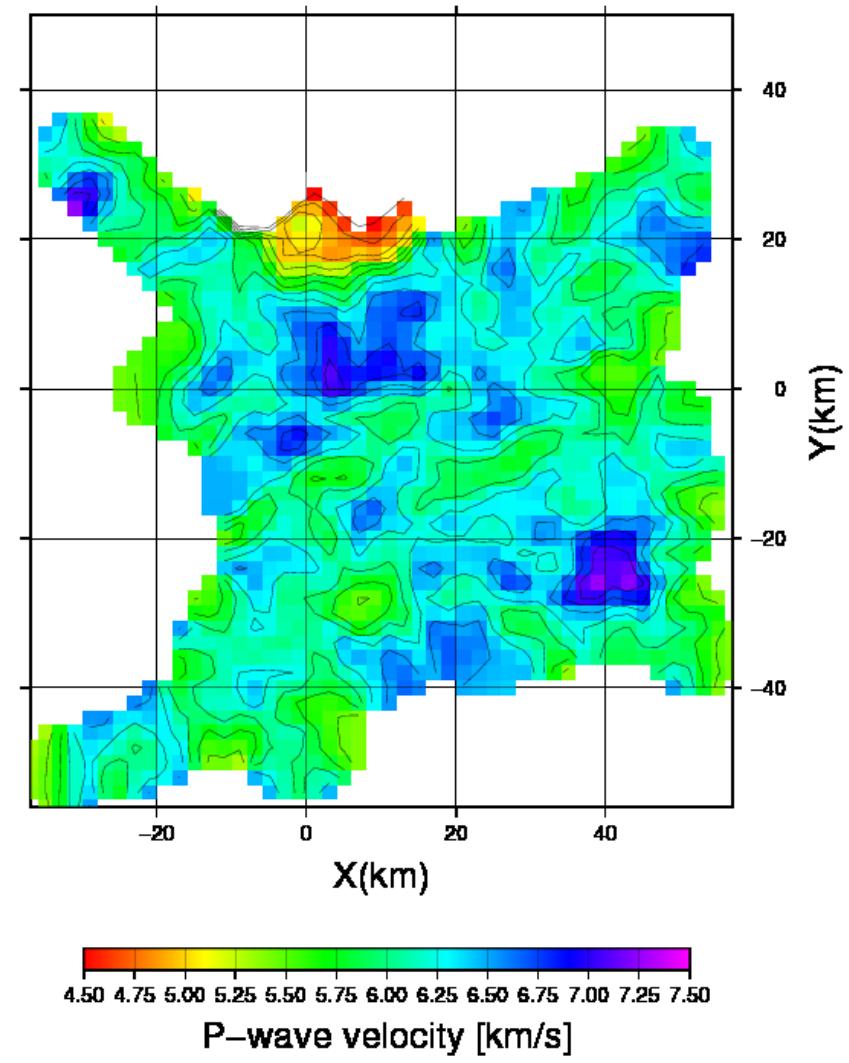
# Horizontal cross-sections in a depth of 4 km

$$\begin{bmatrix} g_1(m_1) \\ g_2(m_2) \\ g_r(m_1, m_2) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} \leftarrow \begin{bmatrix} \Delta t \approx 0.1 \text{ s} \\ \Delta g \approx 0.4 \text{ mGal} \\ \lambda \approx 5 * \text{norm} \end{bmatrix}$$

Depth\_level\_No\_03



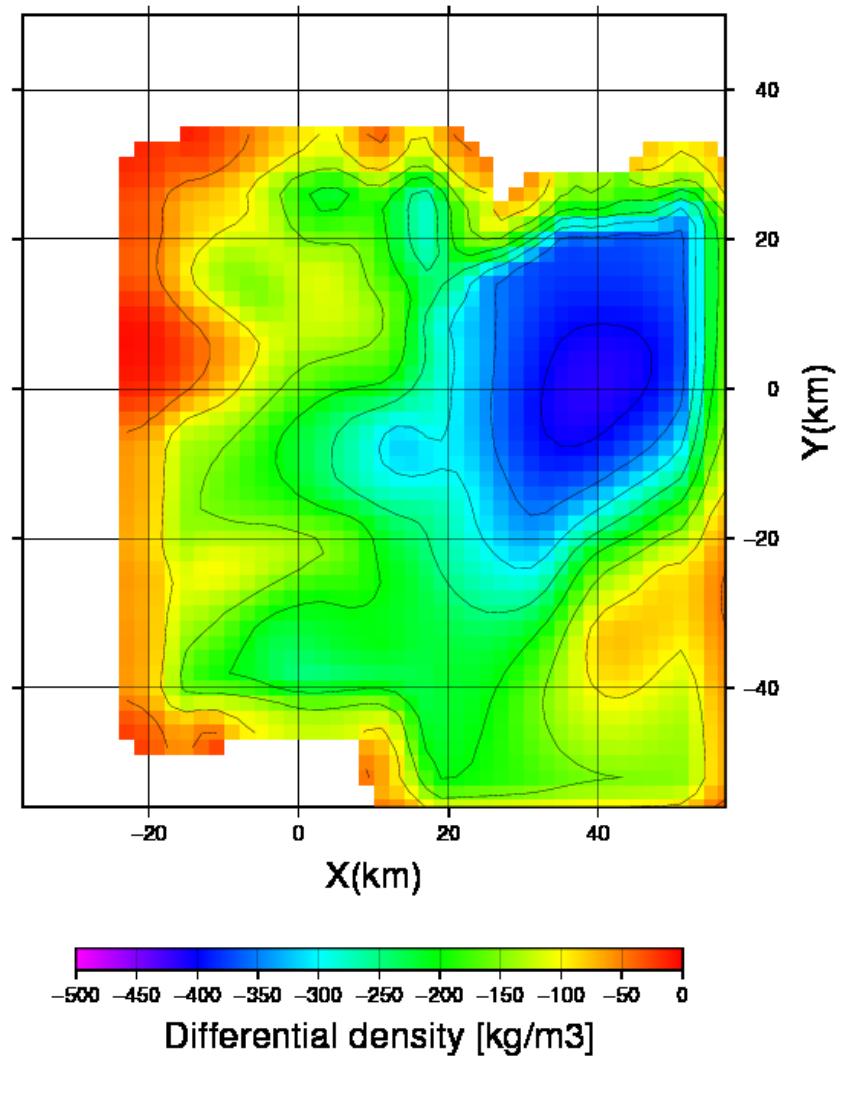
Depth\_level\_No\_03



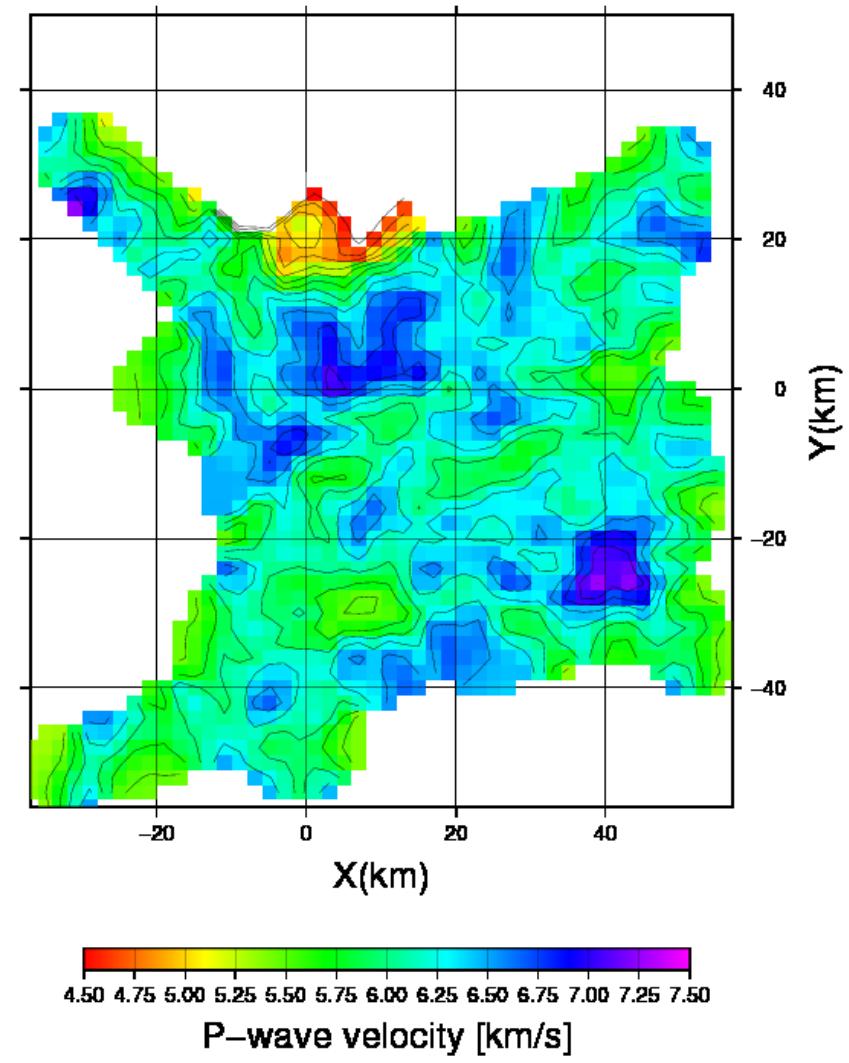
# Horizontal cross-sections in a depth of 4 km

$$\begin{bmatrix} g_1(m_1) \\ g_2(m_2) \\ g_r(m_1, m_2) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} \leftarrow \begin{bmatrix} \Delta t \approx 0.1 \text{ s} \\ \Delta g \approx 0.4 \text{ mGal} \\ \lambda \approx 10 * \text{norm} \end{bmatrix}$$

Depth\_level\_No\_03



Depth\_level\_No\_03



# Joint inversion combining gravity and seismic measurements

- can be linked by cross-gradient constraint, then
- no a-priori relation between model subspaces is required;
- stable solutions to both methods are produced;
- results seem to be reasonable and
- resulting models will be offered for free testing via web.

*Thanks !*

