

## 1. Introduction

Reconstructions of climate variation over recent millennia make an important contribution towards our understanding of climate change, in particular by helping us to place the recent anthropogenically-forced changes in the context of natural variability. Therefore, it is important that we have a sound understanding of the reliability and precision of these reconstructions.

This work has two main aims:

- 1 We investigate the identifiability of the climate state from limited proxy data. That is, we calculate the precision with which it is possible to estimate the climate state based on a handful of heterogeneously-located observations such as the tree ring assembly of Mann et al 2008.
- 2 We test the viability of particle-based approaches for data assimilation over recent millennia (eg Goosee et al 2006) for generating reconstructions. Ensemble size is known to be a critical factor in the success of these methods (Bengtsson et al 2008).

In contrast to the regression-based and frequentist methods which have been widely adopted for millennial reconstructions, here we use an optimal data assimilation methodology to treat the problem as one of Bayesian estimation. The investigations presented here are all based on identical twin experiments within a perfect model paradigm. In other words, pseudoproxy data are generated by a single model 'truth' run, with the aim being to reconstruct (as far as possible) the full climate state from these limited data. Using pseudoproxy data from a model run enables us to validate the method precisely, without concerning ourselves over the additional uncertainties arising from both the interpretation and validity of the real data and also model inadequacies.

## 2. Method

We used a 101-member, 100-year integration of the Earth system model of intermediate complexity, LOVECLIM (Renssen et al 2005). One model at a time is used as the truth, with the other 100 used as the ensemble. In this idealised testing, external forcing is held constant. This allows us to treat all model years exchangeably, generating an effective ensemble size of 10,000. One of the questions we address is how large an ensemble is required for valid and meaningful results.

The data we use are pseudoproxies generated from a model run. In order to have direct relevance to actual reconstructions, however, the location of the proxies is determined by the data collection of Mann et al 2008, gridded onto the LOVECLIM model grid. The variation of the number of grid points (which is rather lower than the number of proxies) through time is plotted on Fig 1. We use an optimistic signal/noise ratio of 1 in these experiments.

The method adopted is a particle filtering approach. That is, the ensemble members are weighted according to their likelihood. For the simple case of independent Gaussian uncertainties on the observations, this is the exponential of the quadratic cost function

$$e^{-0.5 \times \sum_i ((m_i - o_i) / \sigma_i)^2}$$

where i indexes the observations o and the equivalent model outputs m, and sigma is the uncertainty on the observations.

## 3. Results

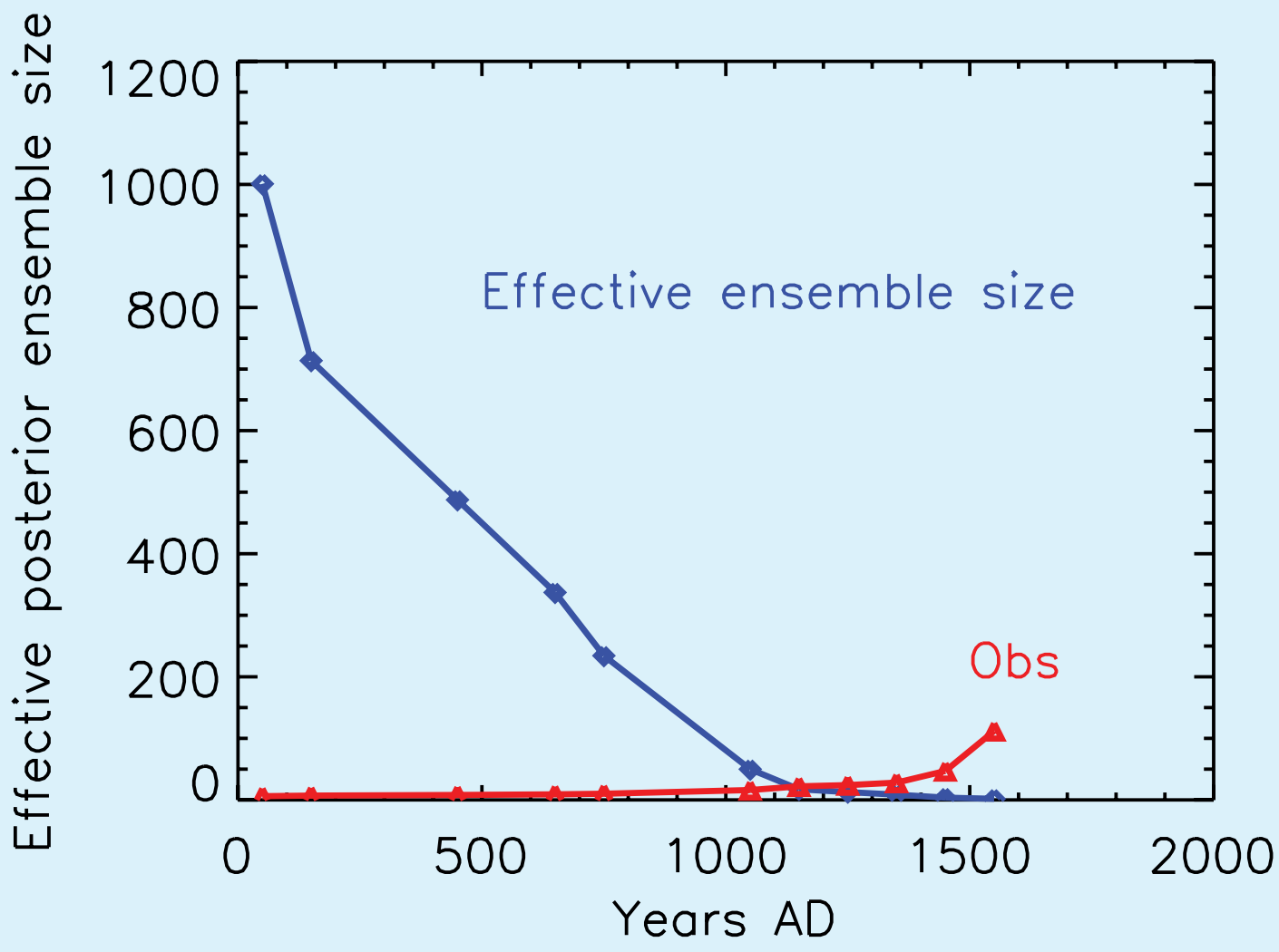


Fig 1. The number of observations grows in time, and the posterior ensemble size (after reweighting) depends on the number of observations. For more than about 15 grid points (ie after 1100AD), the filter has collapsed, indicating that 10,000 ensemble members is not adequate for this problem. In this case, the method automatically chooses the "best" model, but this is not actually a valid estimate of the climate state.

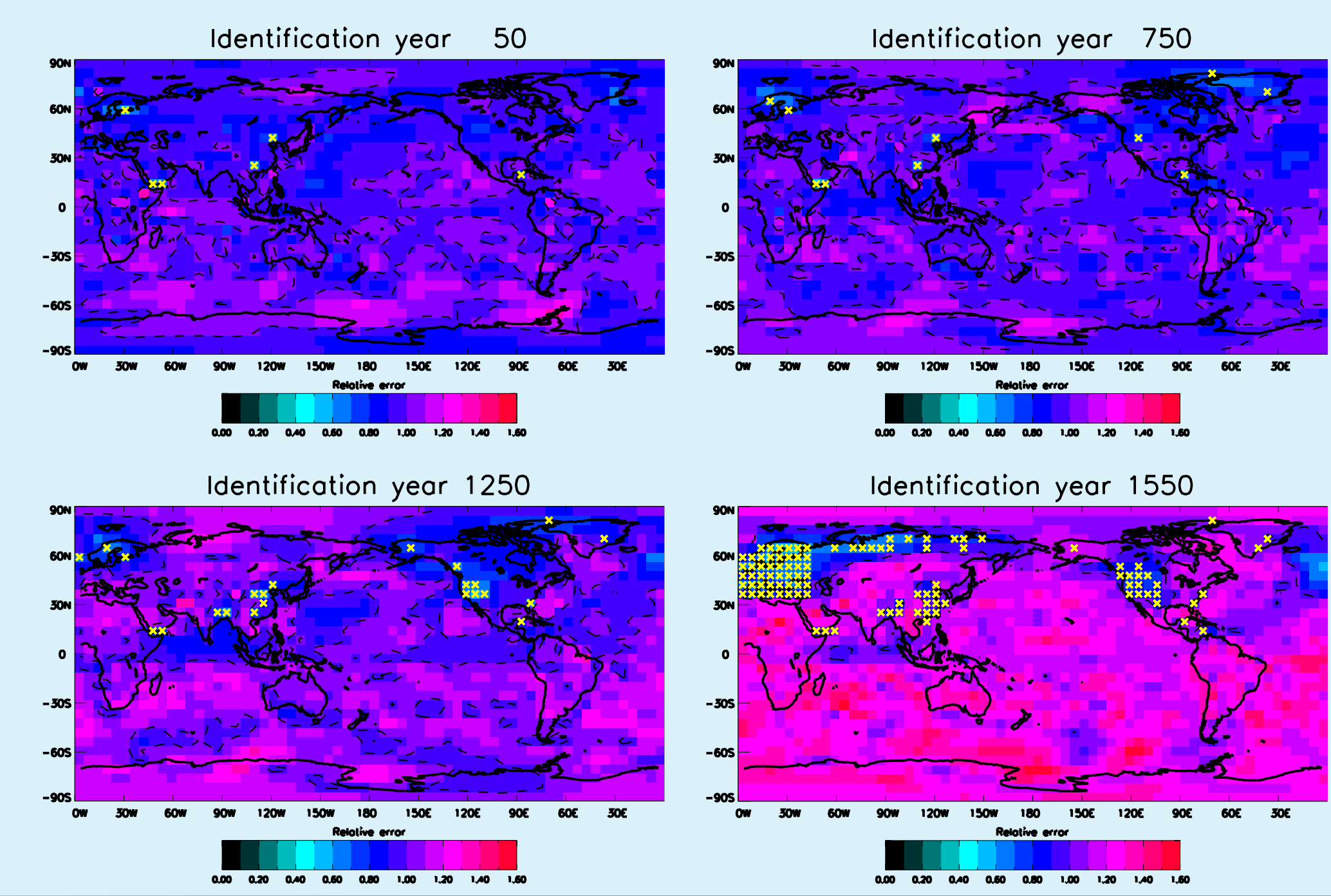


Fig 2. Spatial plots showing the skill of climate reconstruction for 4 different data configurations. Skill/no-skill boundary marked with dotted contour. For few data, there are moderate improvements around the observations, and little effect elsewhere. For many data, the posterior manages to fit these data better than climatology, but at the cost of a marked degradation of fit everywhere else.

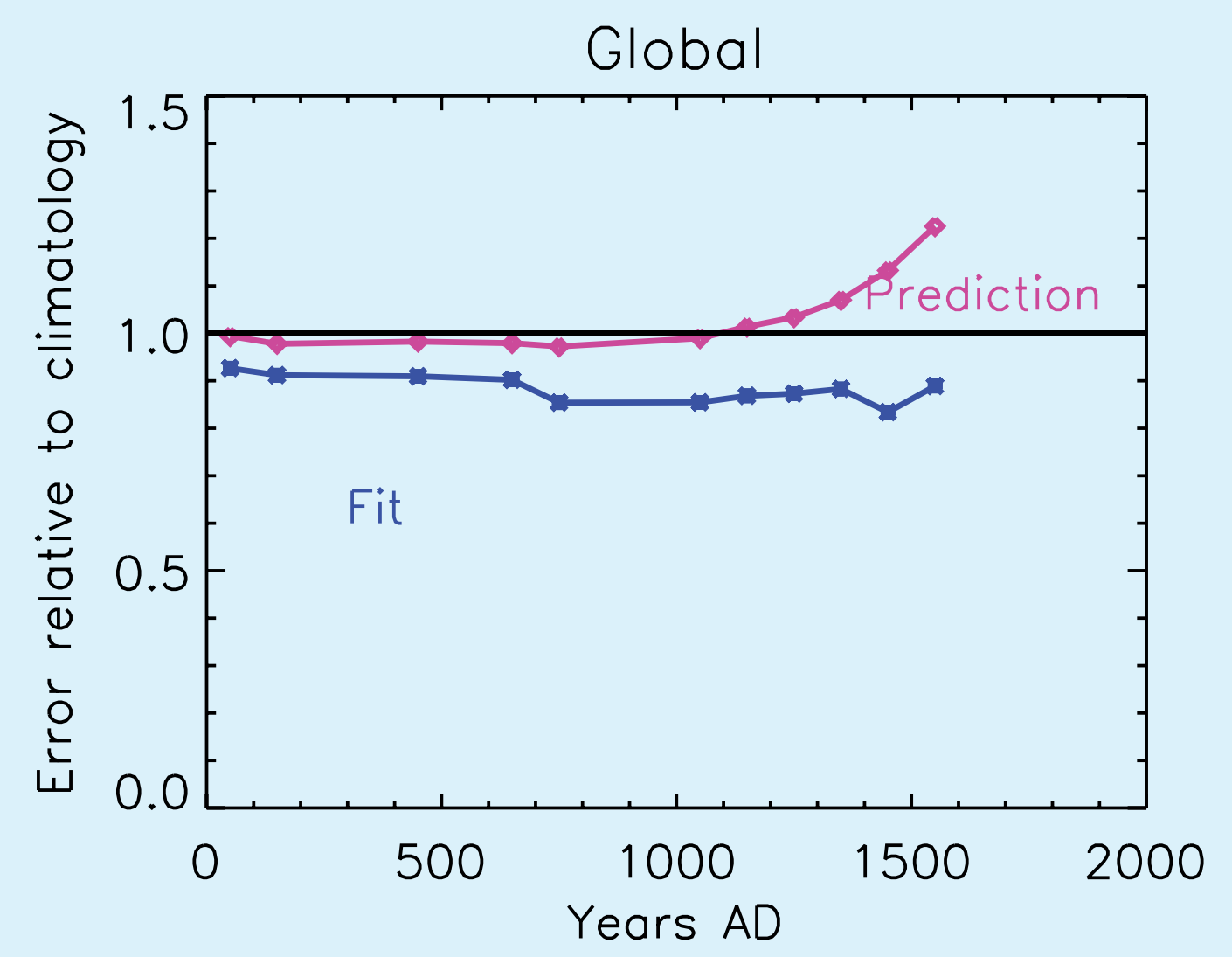


Fig 5. Skill of the method for reconstruction and 1-year prediction of global mean temperature. The reconstruction has a small amount of skill, meaning that the posterior ensemble mean is slightly more accurate than the climatological mean. However the improvement is rather small. The 1-year prediction is actually worse than climatology for the post-1100 estimates, which is further evidence of filter collapse.

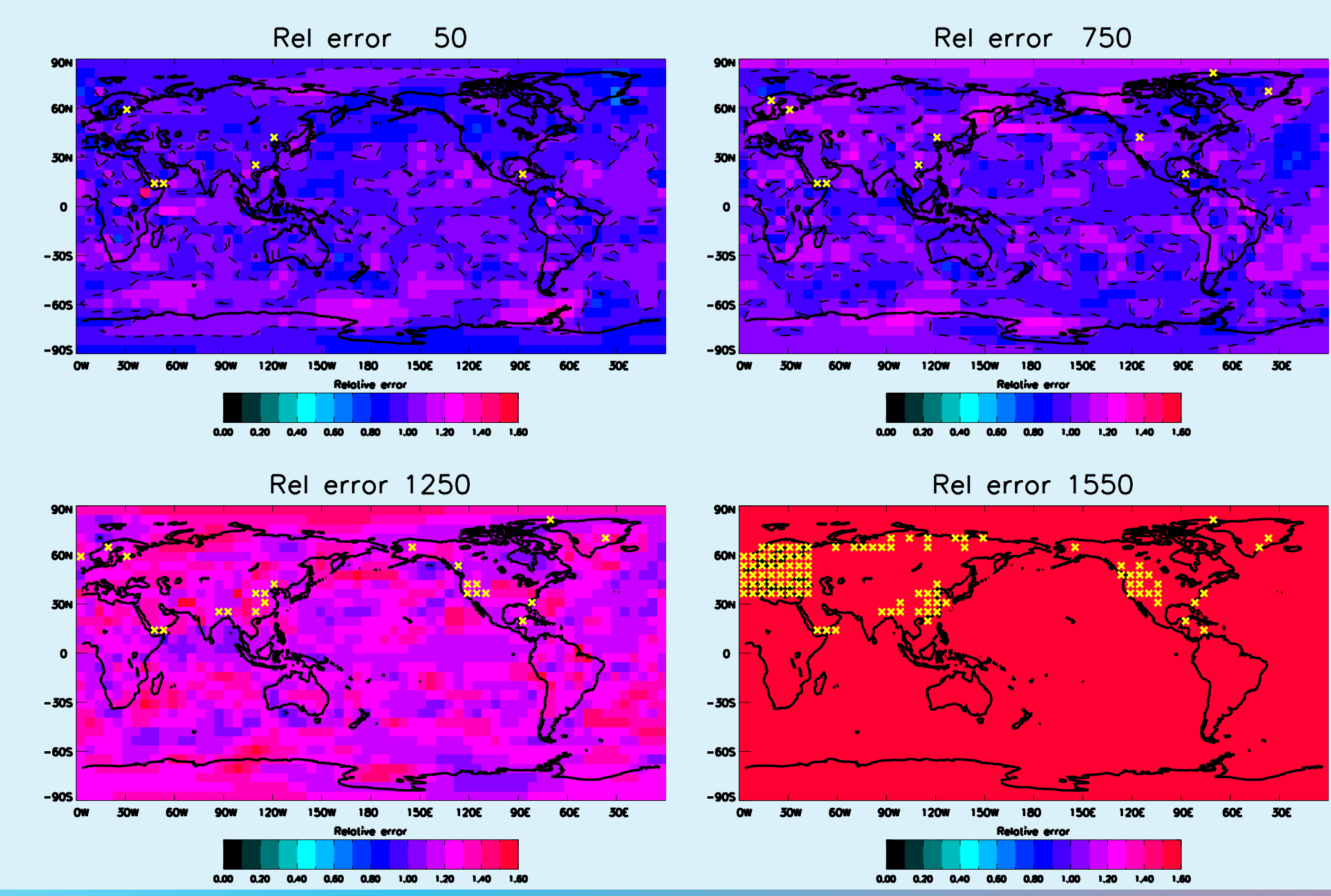


Fig 3. Reliability of posterior ensemble. These figures show how the ensemble width relates to the actual error at each grid point. The ratio should be 1 if the ensemble is reliable. We see that for small numbers of observations, this is indeed the case, indicating that the system is working correctly. However for many obs, the ensemble has collapsed and the truth is well outside the ensemble. The bottom right figure is not a mistake!

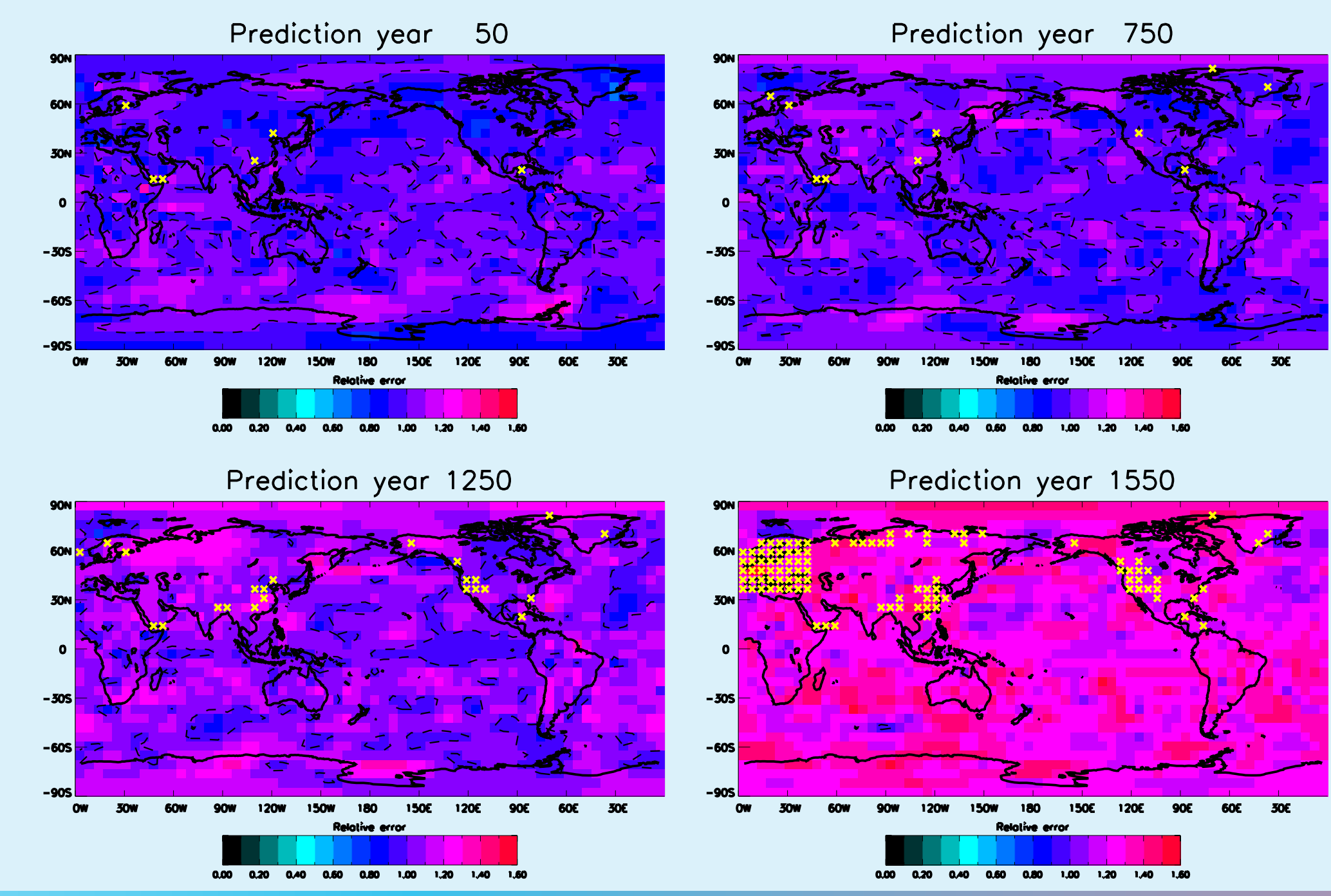


Fig. 4. Prediction skill compared to climatology. At 50AD, the system has forecast skill almost everywhere. At 1550AD, the system is markedly worse than climatology.

## 4. Conclusion

The system works correctly for a small number of data, but in this case the posterior is only marginally better than climatology. For more plentiful data, as is found for the last millennium, the filter collapses and has negative skill almost everywhere on the globe (though small positive skill for reconstruction of the global/hemispheric mean). We have two main conclusions relating to our original goals:

- 1 The climate state cannot be accurately reconstructed with 15 or fewer grid points, although we can do a little better than climatology.
- 2 An ensemble size of 10,000 is inadequate for the last millennium where data are more plentiful, as the ensemble collapses to the best fit model. These results must cast doubt on the viability of particle-based methods as currently implemented, though it may be possible to improve performance with more sophisticated approaches.

## Bibliography

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