Optimizing the model resolution in the inversion of geodetic data

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Outline

- Previous work on this topic
- Aim of this work
- ~~Overview on the inversion techniques~~
- Algorithm description and rationale
- Test cases (Yushu, Christchurch, Tohoku)
- Conclusions and further developments

Previous work (1/2)

- **Sagiya, T. & Thatcher, W.,** 1999. Coseismic slip resolution along a plate boundary megathrust: The Nankai Trough, southwest Japan, *J. Geoph. Res.*, 104(B1), 1111-1129
- **Pritchard, M.E., Simons, M., Rosen, P., Hensley, S. & Webb, F.H.**, 2002. Co-seismic slip from the 1995 July 30 M_w=8.1 Antofagasta, Chile, earthquake as constrained by InSAR and GPS observations, *Geophys. J. Int.*, 150, 362-376.
- **Wright, T.J., Lu, Z. & Wicks, C.**, 2003. Source model for the Mw 6.7, 23 October 2002, Nenana Mountain Earthquake (Alaska) from InSAR, *Geophys. Res. Lett.*, 30(18), 1974, doi:10.1029/2003GL018014.
- **Funning, G.J., Parsons, B. & Wrigth, T.J.**, 2005. Surface displacements and source parameters of the 2003 Bam (Iran) earthquake from Envisat advanced synthetic aperture radar imagery, *J. Geophys. Res.*, 110, B09406, doi:10.1029/2004JB003338
- **Simons, M., Fialko, Y & Rivera, L.**, 2002. Coseismic Deformation from the 1999 M_w 7.1 Hector Mine, California, Earthquake as Inferred from InSAR and GPS Observations, *Bull. Seismol. Soc. Am.*, 92(4), 1390-1402.
- **Fialko, Y., Sandwell, D., Simons, M. & Rosen, P.**, 2005. Three-dimensional deformation caused by the Bam, Iran, earthquake and the origin of shallow slip deficit, *Nature*, 435, 295-299, doi:10.1038/nature03425.

Previous work (2/2)

- **Lohman, R. & Simons, M.**, 2005. Some thoughts on the use of InSAR data to constrain models of surface deformation: Noise structure and data downsamplig, *Geochem. Geophys. Geosyst.*, 6, Q01007, doi:10.1029/2004GC000841
- **Biggs, J., Bergman, E., Emmerson, B., Funning, G., Jackson, J., Parsons, B. & Wright, T.J.**, 2006. Fault identification for buried strike-slip earthquakes using InSAR: The 1994 and 2004 Al Hoceima, Morocco earthquakes, *Geophys., J. Int.*, 166, 1347-1362, doi: 10.1111/j.1365-246X.2006.03071.x
- **Page, M.T., Custódio, S., Archuleta, R.J. & Carlson, J.M.**, 2009. Constraining earthquake source inversions with GPS data: 1. Resolution-based removal of artifacts, *J. Geophys. Res.*, 114, B01314, doi:10.1029/2007JB005449.
- **Custodio, S., Page, M. T. & Archuleta, R.J.**, 2009. Constraining earthquake source inversions with GPS data: 2. A two-step approach to combine seismic and geodetic data sets, *J. Geoph. Res.*, 114, B01315, doi:10.1029/2008JB005746.
- **Cheloni, D., D'Agostino, N., D'Anastasio, E., Avallone, A., Mantenuto, S., Giuliani, R., Mattone, M., Calcaterra, S., Gambino, P., Dominici, D., Radicioni, F. & Fastellini, G.**, 2010. Coseismic and initial post-seismic slip of the 2009*M*w 6.3 L'Aquila earthquake, Italy, from GPS measurements, *Geophys. J. Int.*, 181, 1539-1546, doi: 10.1111/j.1365-246X.2010.04584.x.

Value and limit of previous work

- The resolution is assessed by means of a checkerboard test
- The model resolution is calculated but the patch dimension is left unchanged or is manually adjusted
- The patch dimension is automatically changed only as function of the patch depth
- The patch size is changed with the aim of keeping the model resolution matrix “qualitatively” more diagonal
- Given a fault subdivision, data are subsampled to keep the data resolution matrix as diagonal as possible

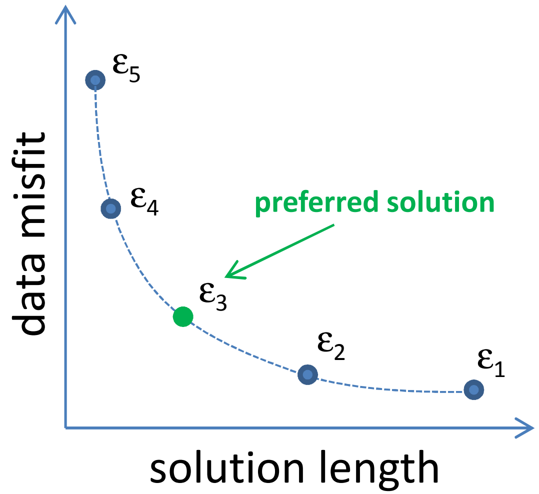
Aim of this work

- Inverting geodetic data with a fault subdivision such that model resolution matrix ≈ identity matrix
  - with a fully automated algorithm
  - without use of *a priori* constraints
- Minimizing the presence of mathematical artifacts in the retrieved slip distribution
- Provide a realistic assessment of the uncertainty

Atzori S. and A. Antonioli, 2011, Optimal fault resolution in geodetic inversion of coseismic data, *Geophys. J. Int.* (2011) **185,** 529–538, doi: 10.1111/j.1365-246X.2011.04955.x

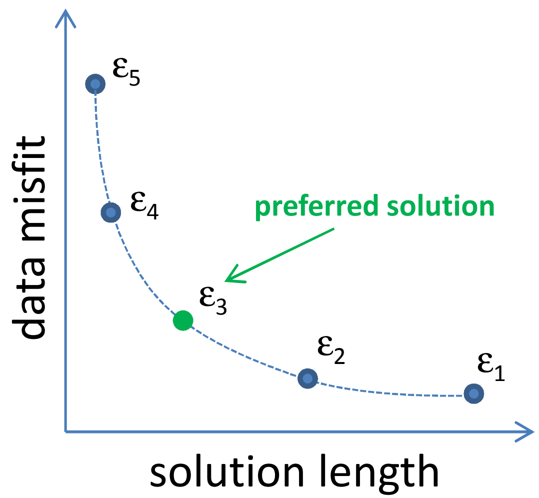
Standard approach for linear inversion

- Set up the linear system with a predefined number of patches
- Set a value for ε (damping)
- Make the inversion (with *a priori* constraints)
- Plot data fit vs. model roughness
- Define the preferred solution



Proposed approach for linear inversion

- ~~Set up the linear system with a predefined number of patches~~
- Set a value for ε
- Find the optimized fault subdivision for the given ε
- Make the inversion (without a priori constraints)
- Plot the data fit and the model roughness
- Define the preferred solution



The model resolution matrix

$$R = G^{-g} G$$

Resolution value: the value from R corresponding to a given parameter; when R(i,i) = 1 the parameter is well resolved

Resolution: the ability of the system in resolving model details

We don't have the maximum resolution when the parameters have the maximum resolution value

- Weighted damped least squares inversion:

$$G^{-g} = (G^T W_e G + \varepsilon W_m)^{-1} G^T W_e$$

$$R = (G^T W_e G + \varepsilon W_m)^{-1} G^T W_e G$$

The “optimal” solution

“optimized” fault subdivision

- the number of patches is maximized
- every patch has R(*i*,*i*) ≈ 1

It’s an iterative algorithm

1. Set a starting coarse subdivision
2. Calculate the model resolution matrix R
3. Identify patches with R(*i*,*i*)<0.99: these are fixed from now on
4. Identify patches with R(*i*,*i*)>0.99: these are candidate for a further subdivision
5. Select <u>a subset</u> to be further subdivided
6. Split in two parts the selected patches
7. Continue from point 2.

Factors affecting the resolution

- Patch dimension
- Depth of the patch
- Data coverage
- Proximity to other patches

Coefficient calculation

- Coefficient accounting for the patch depth:

$$C_1(i) = e^{-\frac{d_i}{D} k_d}$$

d_i depth of the patch

D fault bottom

k_d empirical parameter (3÷5)

- Coefficient accounting for data coverage:

$$C_2(i) = \frac{\mathrm{MIN}\{dist_i\}}{dist_i}$$

$dist_i$ planimetric distance patch/closest observed point

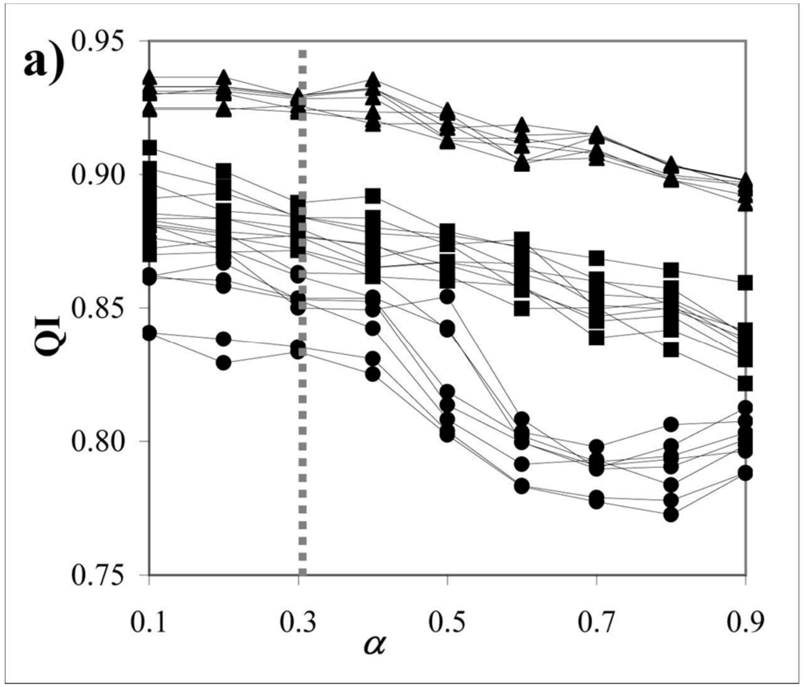
- Coefficient accounting for patch proximity:

$$C_3(i) = \frac{\sum_{j=1}^{M} dist_{i,j} \cdot res_j}{\sum_{j=1}^{M} dist_{i,j}}$$

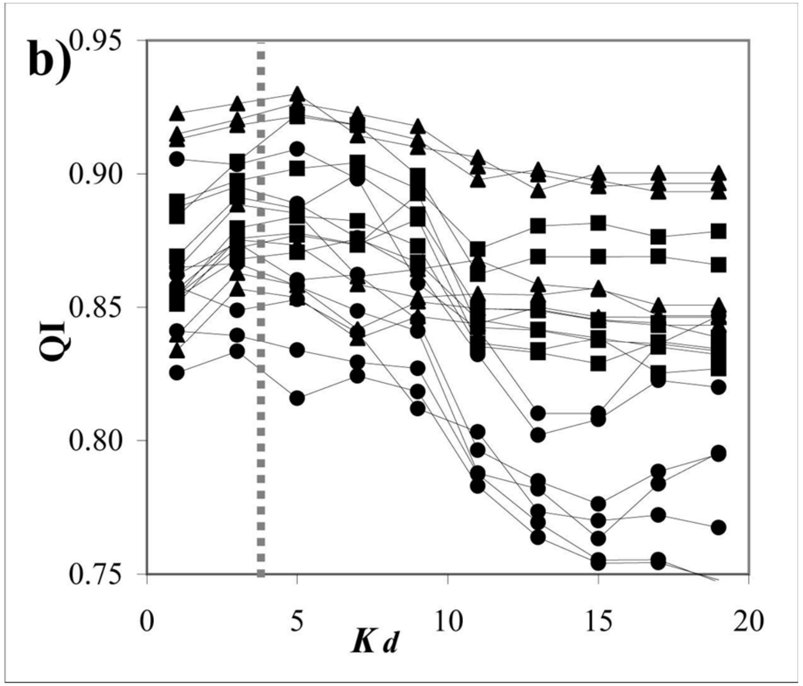
$dist_{ij}$ distance between patch i and j

res_j resolution value of patch j

Empirical parameters



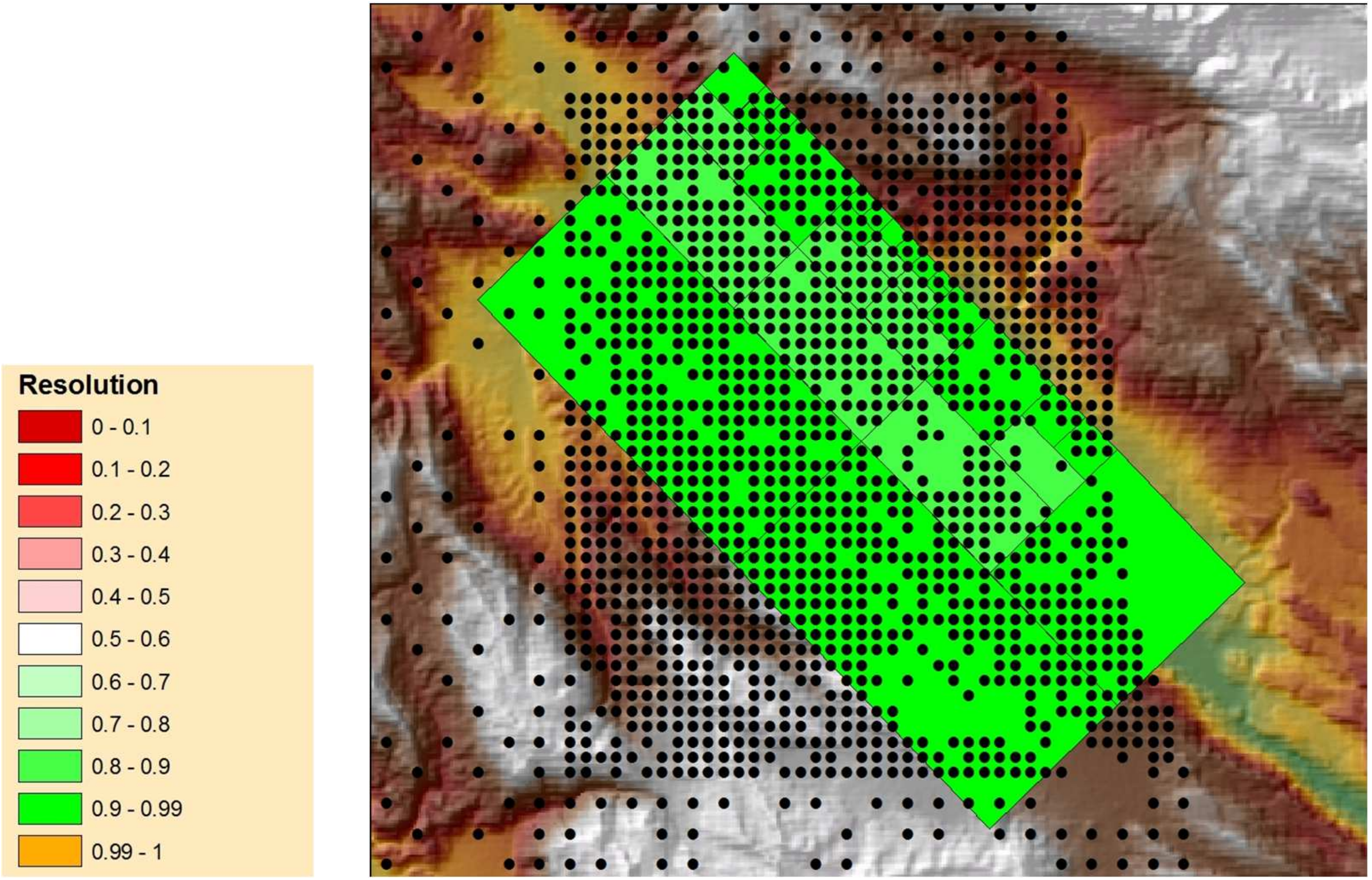
Percentage of patches to be divided



Empirical parameters accounting for the patch depth

Algorithm iteration

Here there was an animation to show the evolution of the fault subdivision, not shown in this pdf version

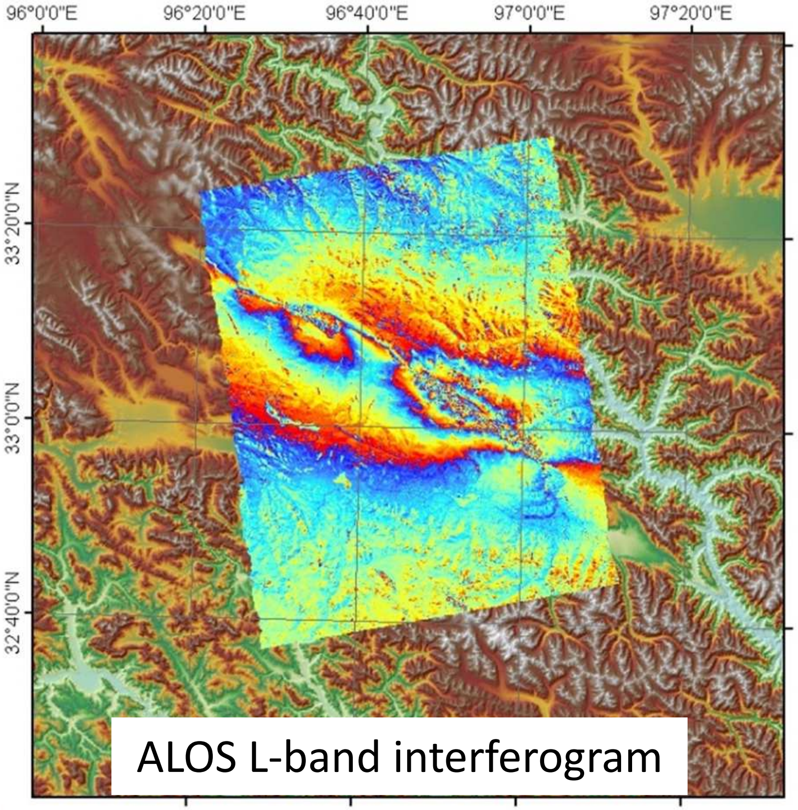


The 2010 Yushu (China) earthquake

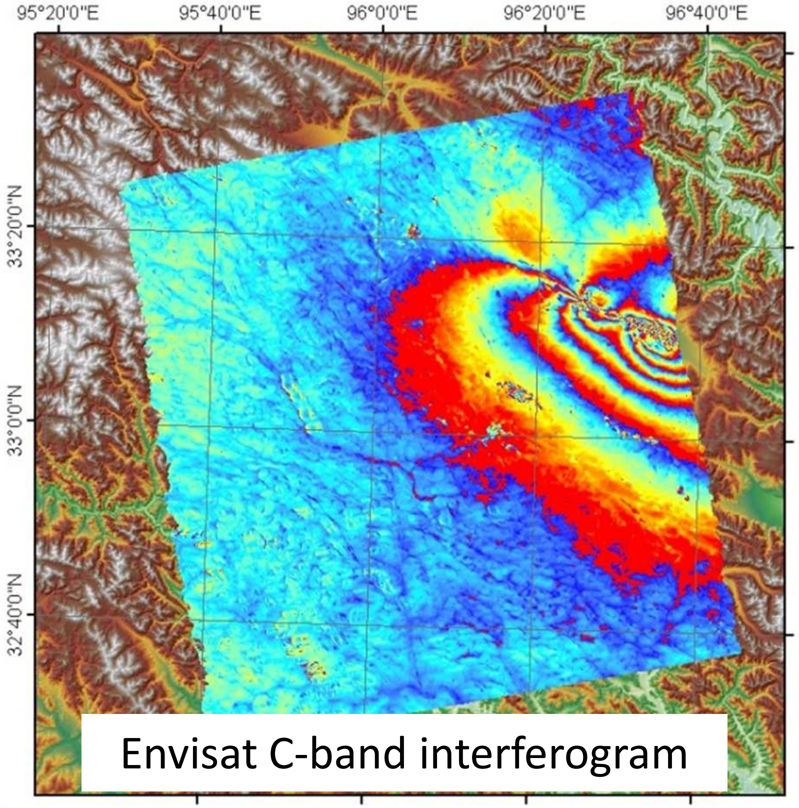
April, 13, 2010

M_w 6.9

Strike 120, Dip 90, Rake -13

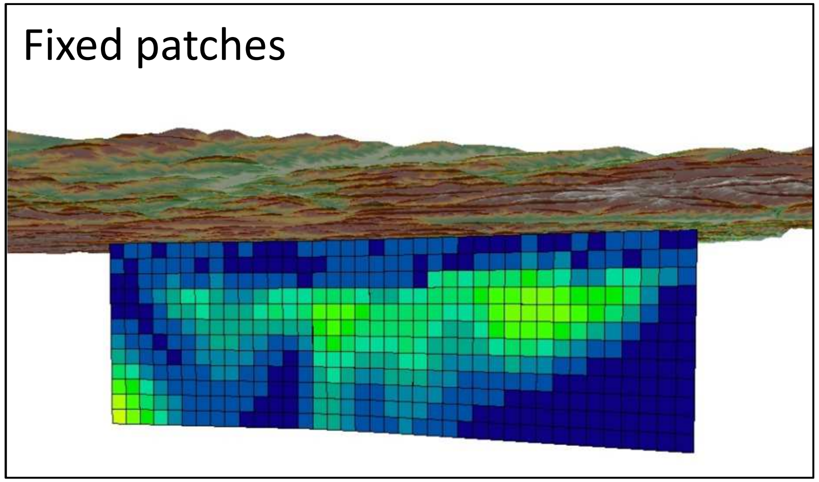


ALOS L-band interferogram



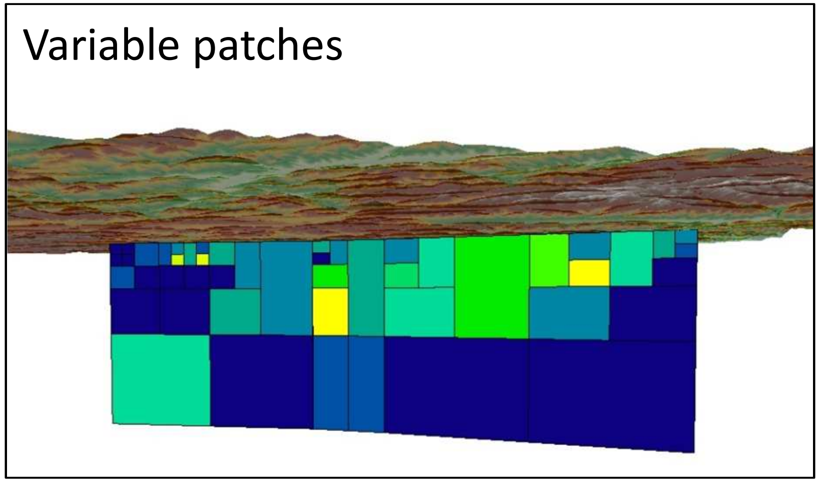
Envisat C-band interferogram

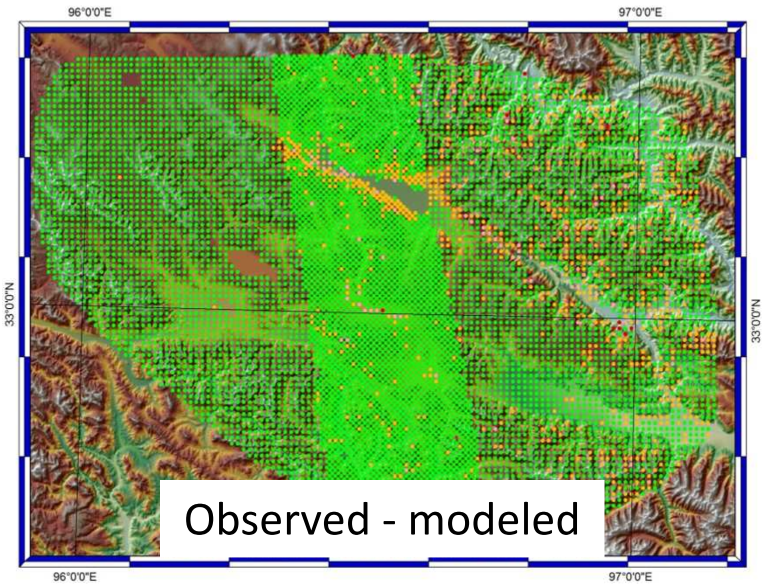
Yushu earthquake models



Slip (cm)

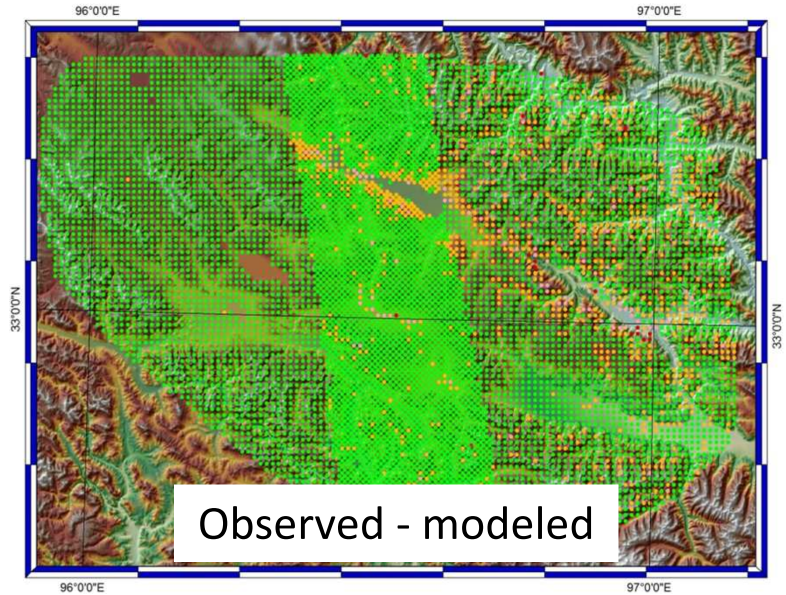
- 0
- 0 - 25
- 25 - 50
- 50 - 75
- 75 - 100
- 100 - 125
- 125 - 150
- 150 - 175
- 175 - 200
- 200 - 225
- 225 - 250
- 250 - 275
- 275 - 300
- > 300





Residuals (m)

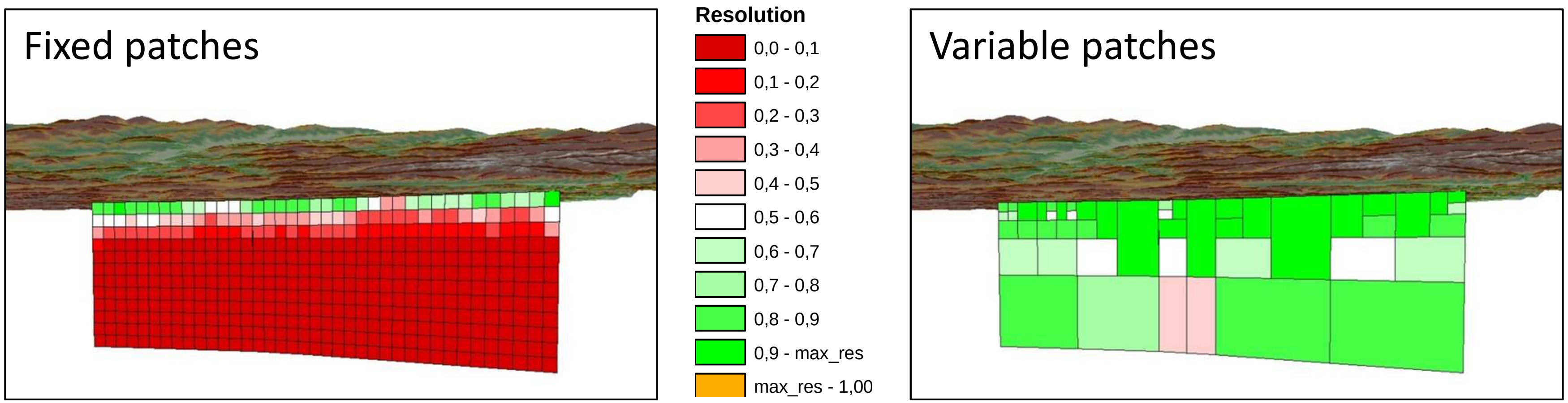
- -1.00 - -0.10
- -0.10 - -0.05
- -0.05 - -0.02
- -0.02 - 0.02
- 0.02 - 0.05
- 0.04 - 0.10
- 0.10 - 1.00



Yushu earthquake, model resolution

Model resolution matrix

$$R = G^{-g}\,G$$

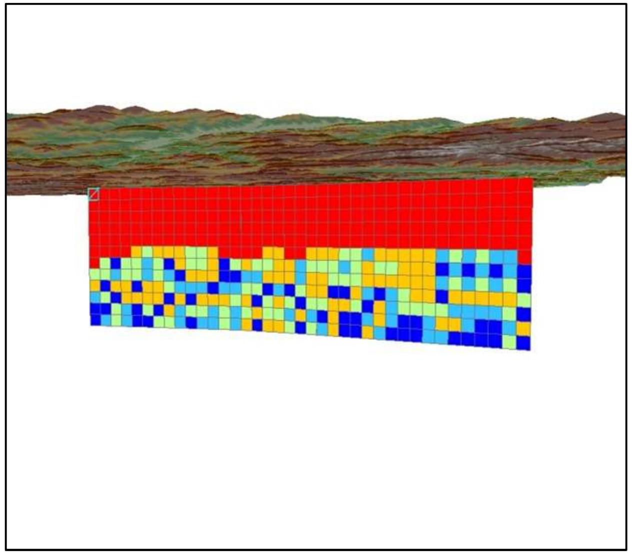


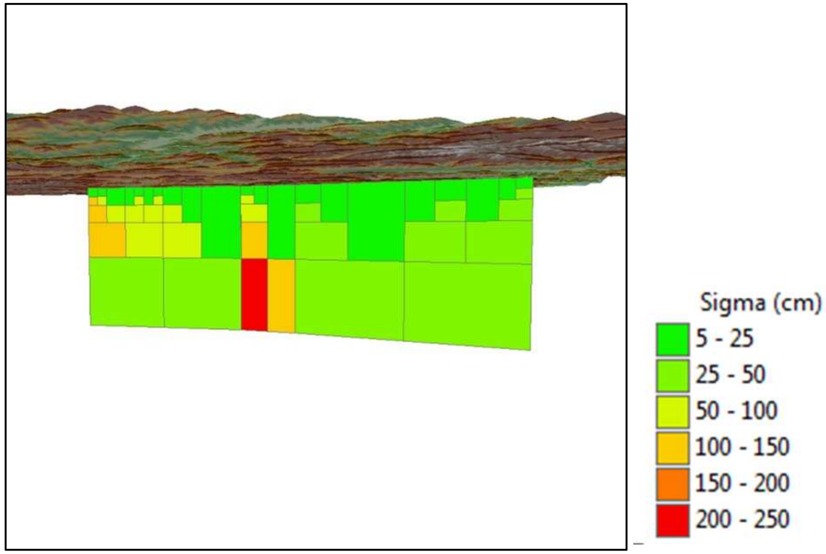
p.s. A vertical (transcurrent) fault is the worse geometry in terms of resolution

Yushu earthquake, model uncertainty

Uncertainty propagation

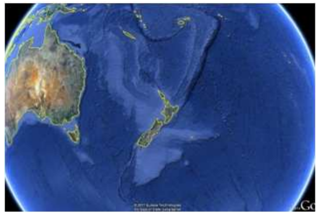
$$cov[m] = G^{-g}\, cov[d]\, G^{-gT}$$





- unrealistic values
- very high trade-offs

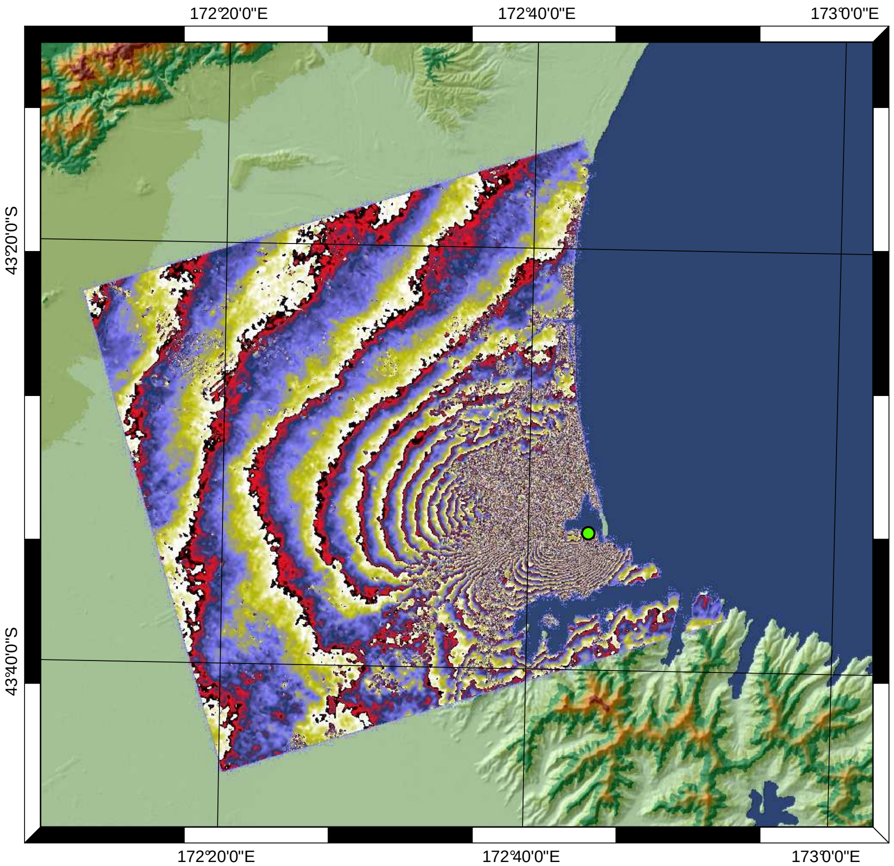
Christchurch (NZ) 2011 event

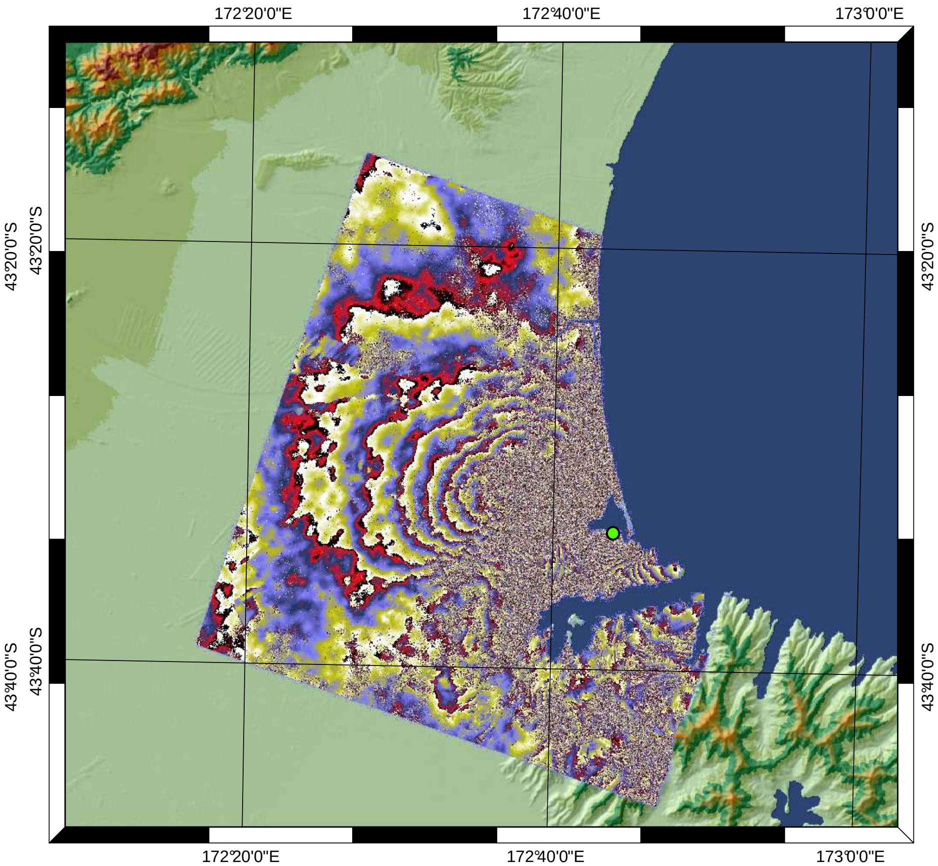


February, 21, 2011

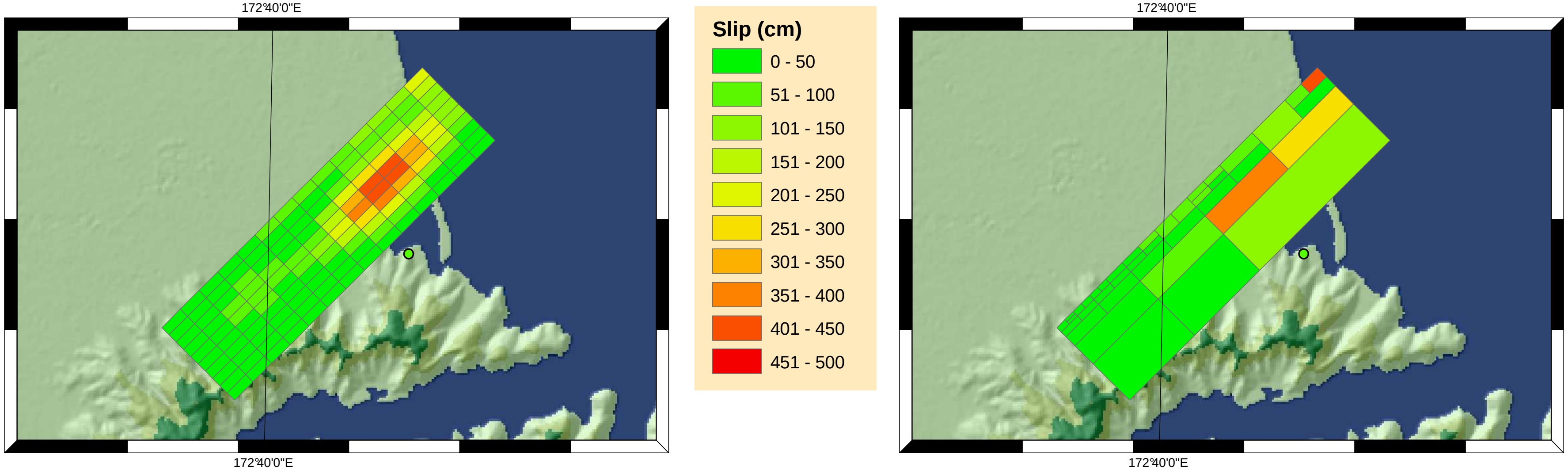
M_w 6.1

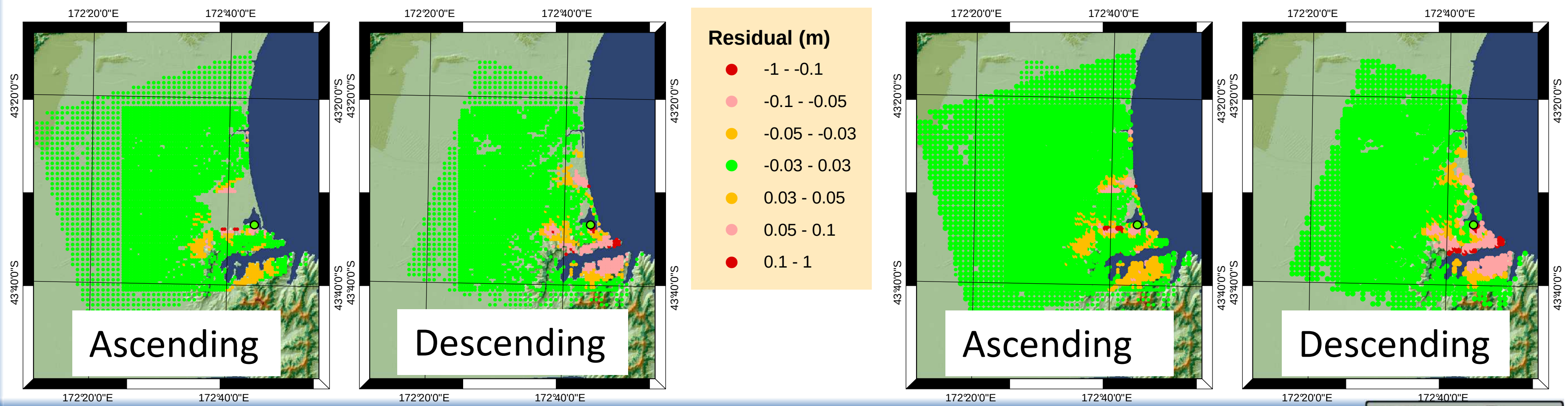
Strike 45, Dip 67, Rake 145



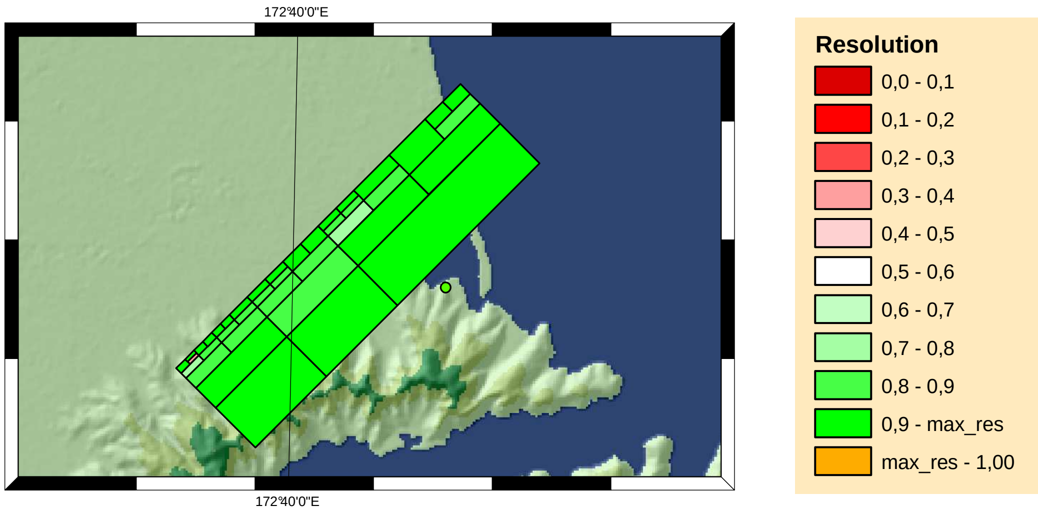


Christchurch models

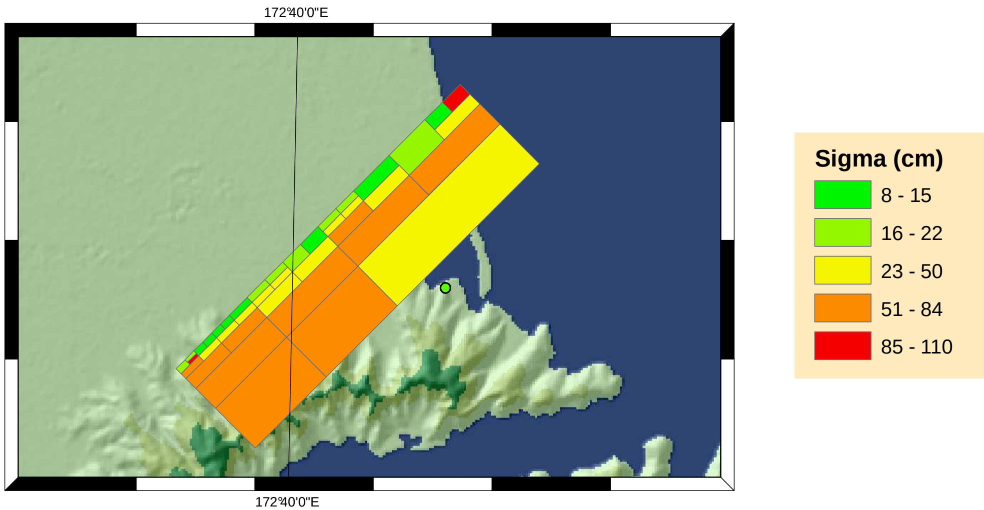




Resolution and uncertainty

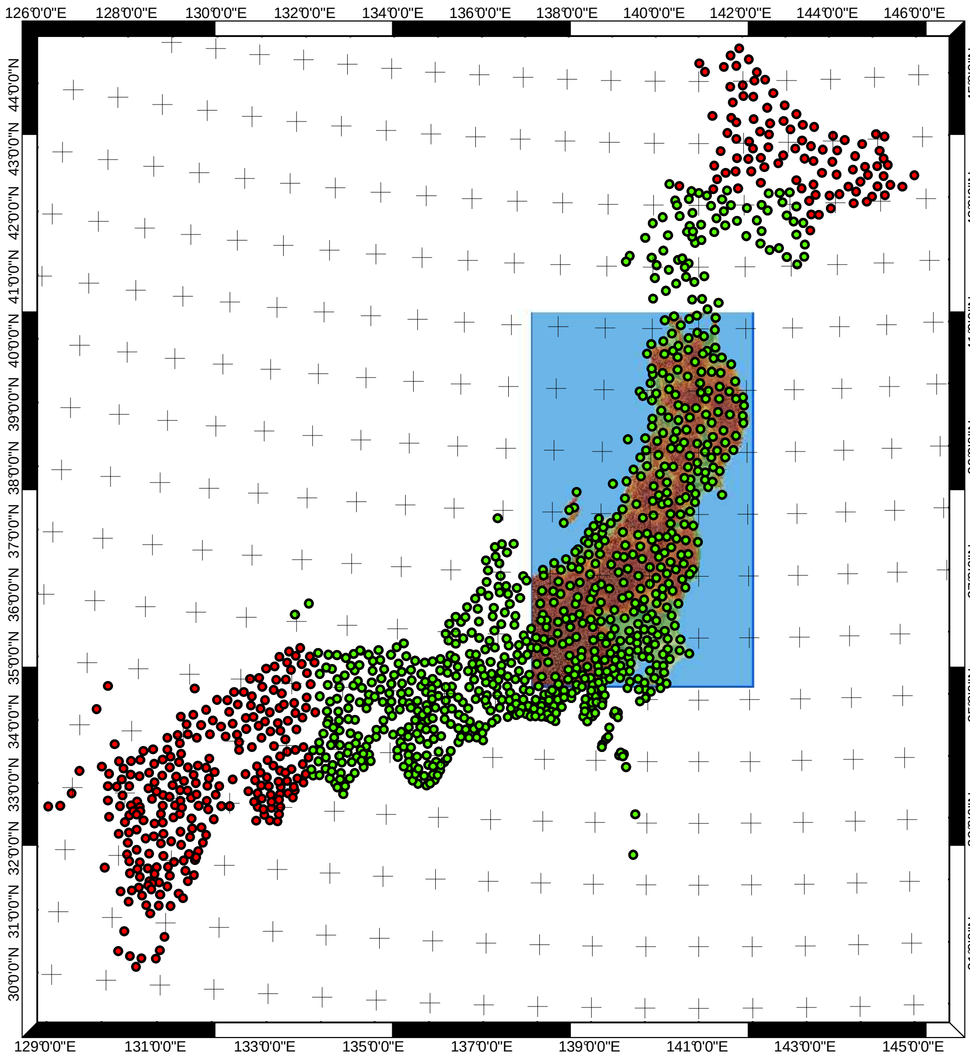


The values of the model resolution matrix are close to 1, regardless their depth



The uncertainty of the model parameters are realistic and trade-offs between adjacent patches are minimized

Tohoku (Japan) 2011 event



March, 11, 2011

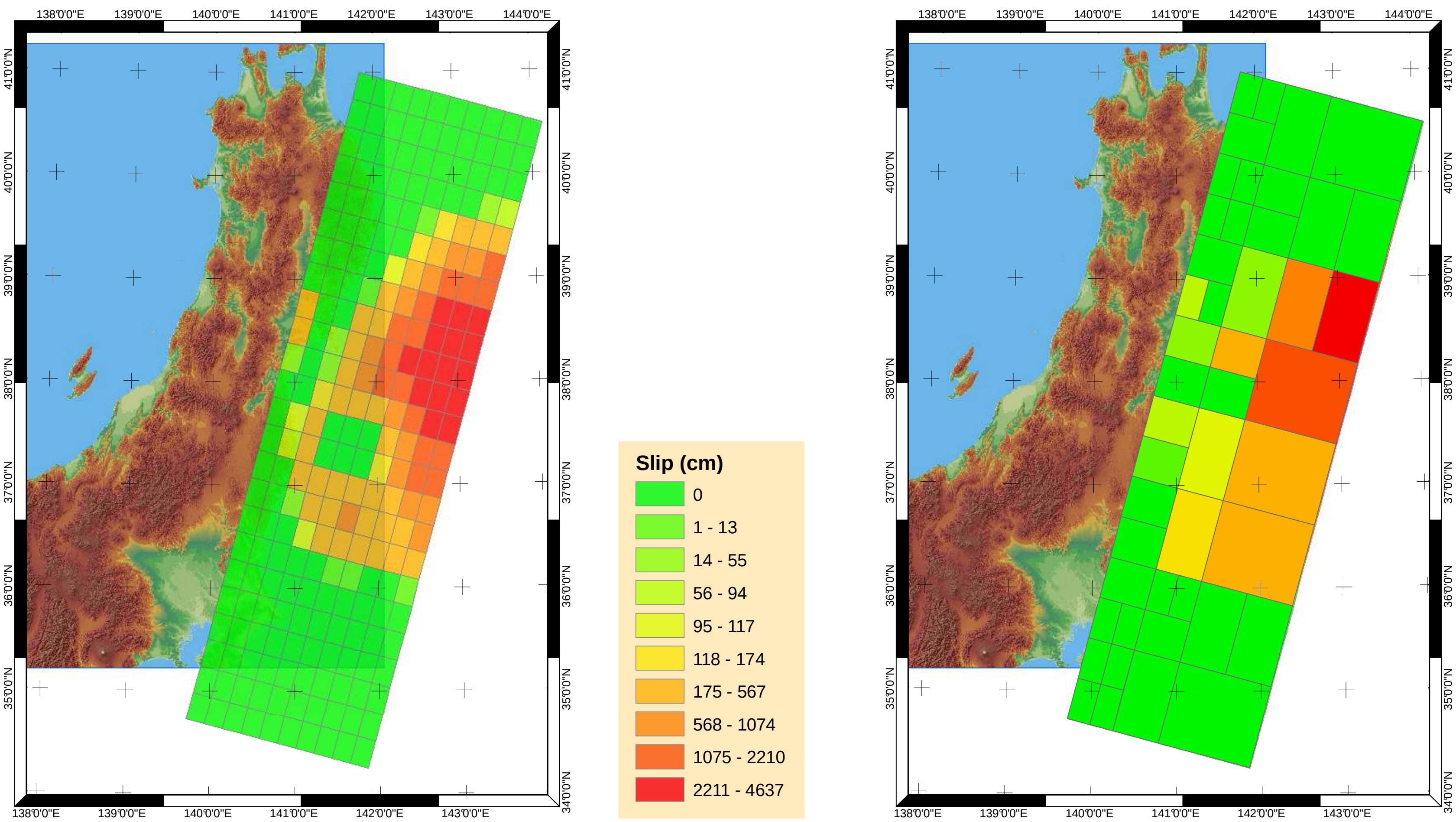
M_w 9,0

Strike 195, Dip 15, Rake 80

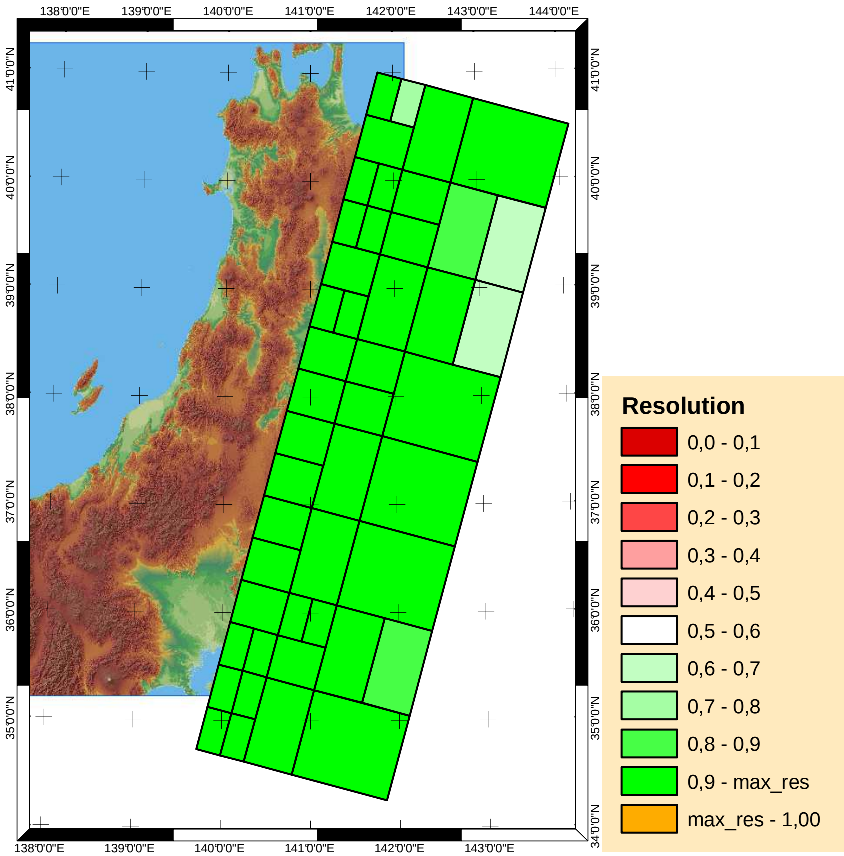
1237 GPS stations available(*);
827 used for modeling.

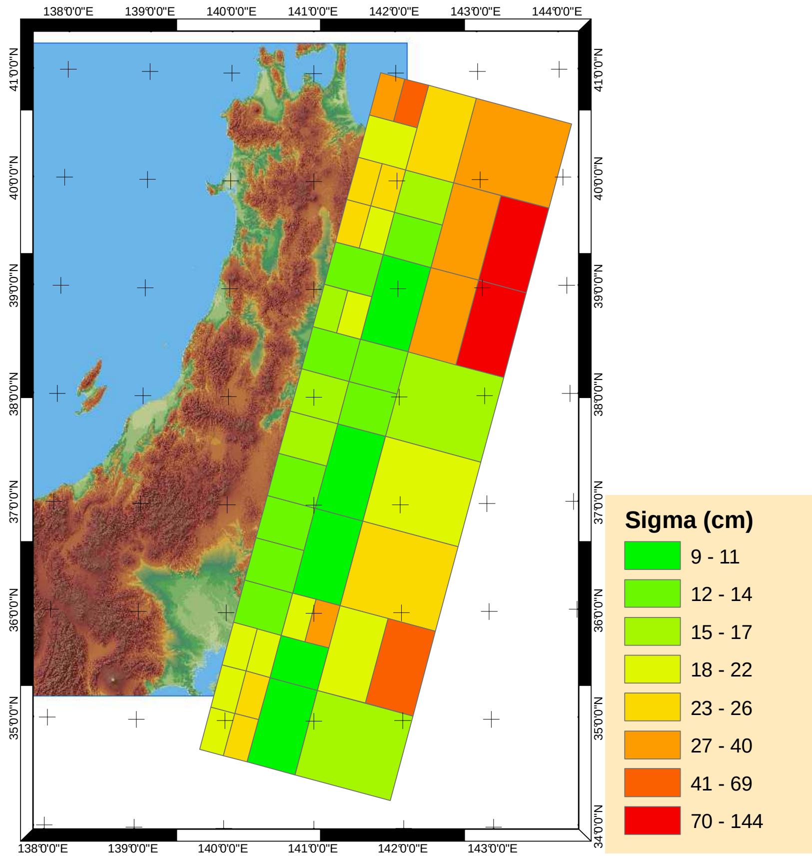
(*) by courtesy of Nicola D'Agostino (INGV)

Dislocation models



Resolution and uncertainty





Achieved goals and further developments

- Implementation of a fully automated algorithm, tested for all the fault geometries and mechanisms
- Achievement of unprecedented optimization of the model resolution matrix
- Avoid the use of *a priori* constraints in the slip retrieval
- Realistic assessment of the model uncertainty

- Test the impact of this algorithm with variable rake models

Thank you!