A non-Gaussian decomposition of GRACE-derived time-variable gravity signals, using Independent Component Analysis (ICA)

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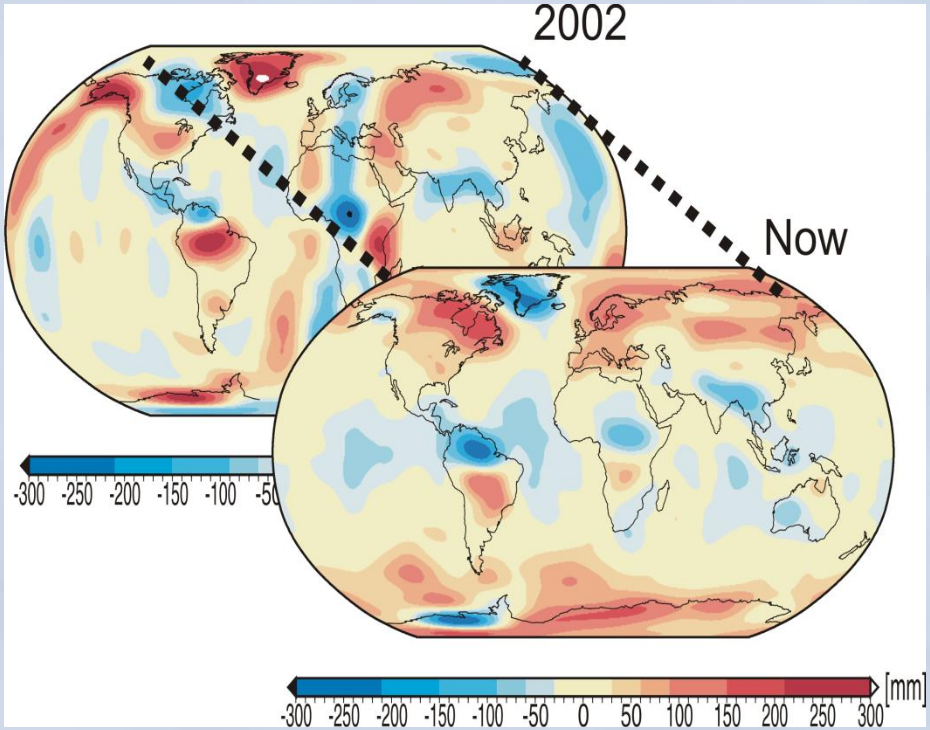
• Temporal information from GRACE

1. Since 2002, GRACE has provided valuable information about mass redistribution within the Earth system.

2. TWS anomalies represent integrated mass over global vertical columns, caused by
   - The Earth's interior
   - Its surface
   - Atmosphere

Challenge: Separation of the observed signals into their original sources



Time series of the Total Water Storage (TWS) maps, derived from the processing of ITG2010 solutions

$$TWS = F(t,s)_{n\times m} = [f_1, f_2, \ldots, f_m]$$

number of solutions

number of grid points

Separation schemes

- Separation of signal and data noise,
  - E.g. **isotropic** (Jekeli, 1981) and **non-isotropic filters** (Kusche, 2007)
  - **Statistical approaches** such as PCA/EOF (e.g. Wouters and Schrama, 2007) and ICA (e.g., Frappart et al., 2010)

- Separation of mass flux patterns from different compartments of the Earth system,
  - **Reduce the unwanted observed quantities** by applying dedicated models e.g. atmosphere, ocean (Flechtner, 2007)
  - **Inversion techniques,** using dynamical theories (e.g. sea level equation) and fitting them to multi-mission data (e.g. GRACE/Jason), (see e.g. Kusche et al., 2011 (talk, Room 18, at 09:30), Rietbroek et al., 2011 (Poster Hall XL Nr. 50)), for estimation of GIA (e.g. Wu et al., 2010)

- Identification of physically meaningful signals within the same compartment, e.g. **PCA/EOF**, **REOF**, **CEOF**, **MSSA** (based on second order statistics) and **ICA** (based on higher order statistics)

Statistical Pattern Extraction

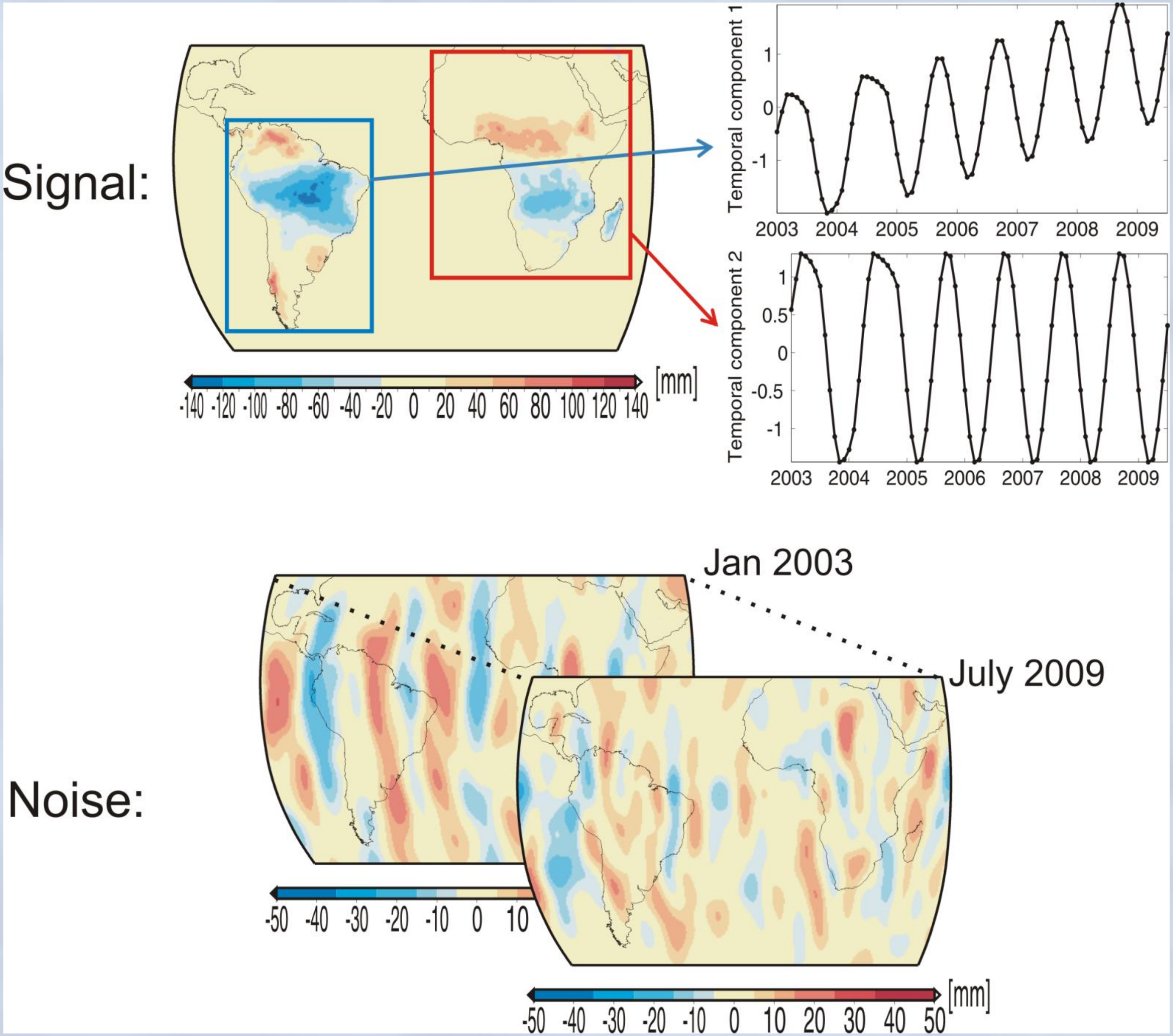
- **PCA**
  - is the most widely used method which works based on eigenvalue decomposition (Lorenz,1956).

$$F(t,s)_{n\times m} = \sum_{k=1}^{m} PCs_k(t)\,EOF_k(s) = P\,E^T$$

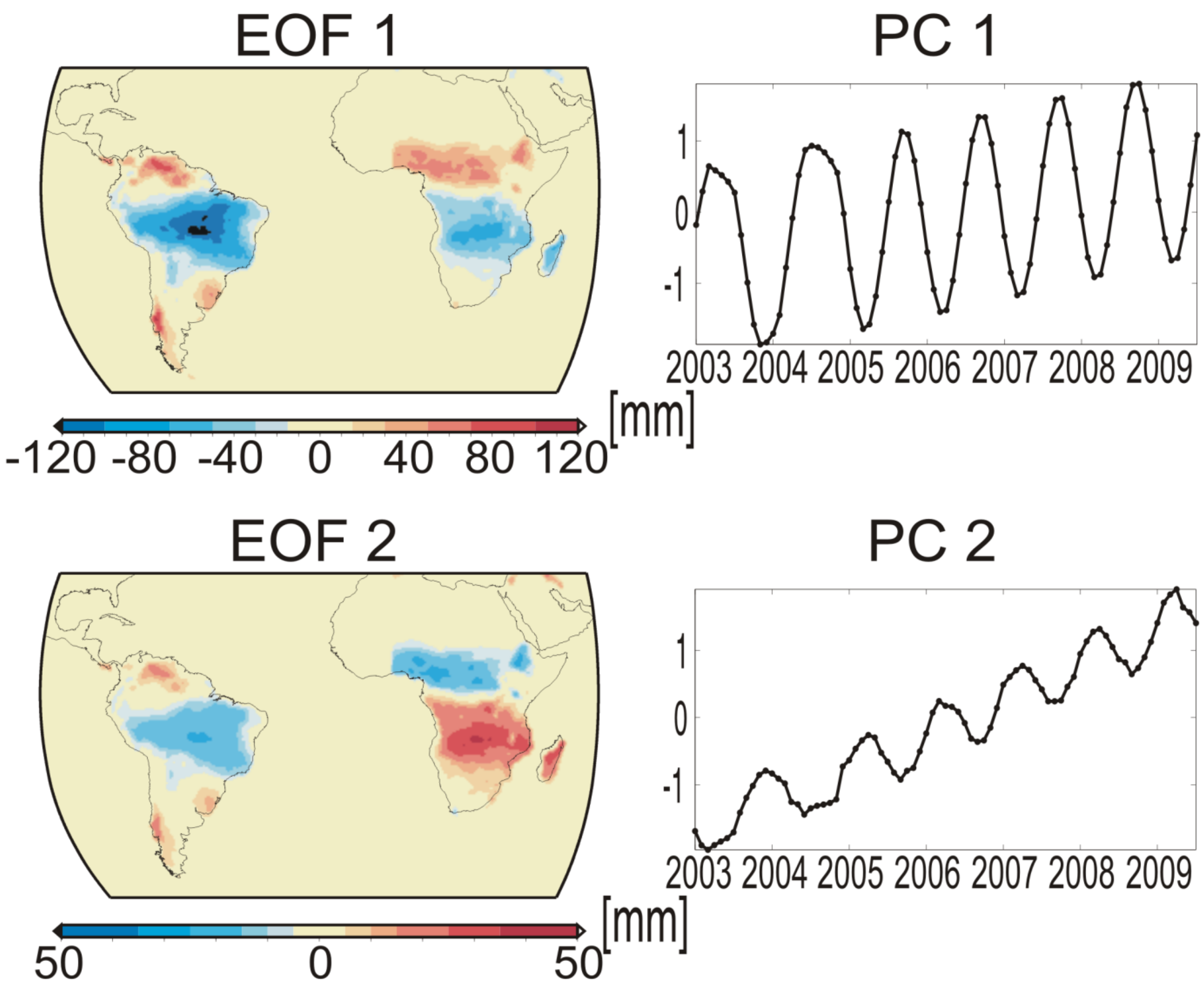
- **Benefits:**
  - De-correlates the dataset by decomposing it to the orthogonal components.
    - Covariance matrix of any subset of retained components is always diagonal.
  - Captures a maximum variability within a few components.

- **Limitation:**
  - Physical process are not necessarily orthogonal.
  - Capturing the maximum amount of variance goes with the 'mixing problem' which might lead to misinterpretation.

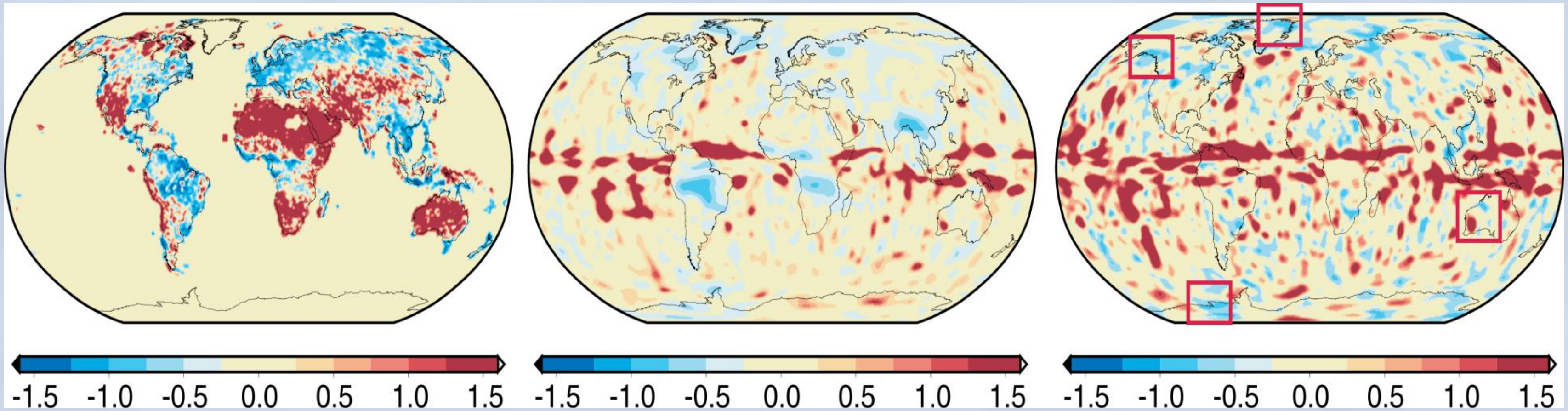
Simulation Status



PCA`s Separation Performance



1. TWS variation is a hydrological parameter associated to physical processes.
   - TWS time series contain a significant level of non-Gaussianity.



Kurtosis of WGHM TWS time series — Kurtosis of GRACE TWS time series — Kurtosis of GRACE TWS after removing trend and annual cycle

- Kurtosis: $$E(x^4)/E(x^2)^2-3\begin{cases}0 & \rightarrow \text{Gaussian}\\ <0 & \rightarrow \text{Sub}-\text{Gaussian}\\ >0 & \rightarrow \text{Super}-\text{Gaussian}\end{cases}$$

⟹ Higher order statistics can be incorporated in the decomposition procedure. (PCA, REOF, CEOF; MSSA and etc. only use the second order statistics)

Why ICA Decomposition?

2. Statistically:

- **Independence** is **stronger** statistical hypothesis than **uncorrelatedness** (e.g. in PCA)
  - Independence implies uncorrelatedness but the reverse is not always true!

⟹ For non-Gaussian signals, maximally independent signals are also likely approximately uncorrelated.

3. Our working hypothesis:

- If different phenomena (**'sources'**) come from different physical processes, they are **statistically mutually independent**.

⟹ **Independent patterns**, are more likely to be related to **'independent physical processes'** than dependent patterns. ⟹ **ICA**

2 Steps ICA algorithm

Step1: Perform PCA, to **decorrelate** the observations.

$$\mathrm{F}(\mathrm{t},\mathrm{s})_{n\times m} = \mathrm{P}\,\mathrm{E}^{T}$$

Step2: Define a suitable **rotation** to optimize an **independence criterion.**

$$\mathrm{F}(\mathrm{t},\mathrm{s})_{n\times m} = \mathrm{P}\,\mathrm{R}\,\mathrm{R}^{T}\mathrm{E}^{T}$$

- **Selected criterion:** fourth order cumulant: if : $\bar{x} = E(x)$

$$C(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4) = E(\bar{\mathrm{x}}_1\bar{\mathrm{x}}_2\bar{\mathrm{x}}_3\bar{\mathrm{x}}_4) - E(\bar{\mathrm{x}}_1\bar{\mathrm{x}}_2)E(\bar{\mathrm{x}}_3\bar{\mathrm{x}}_4) - E(\bar{\mathrm{x}}_1\bar{\mathrm{x}}_3)E(\bar{\mathrm{x}}_2\bar{\mathrm{x}}_4) - E(\bar{\mathrm{x}}_1\bar{\mathrm{x}}_4)E(\bar{\mathrm{x}}_2\bar{\mathrm{x}}_3)$$

(Cardoso, 1998)

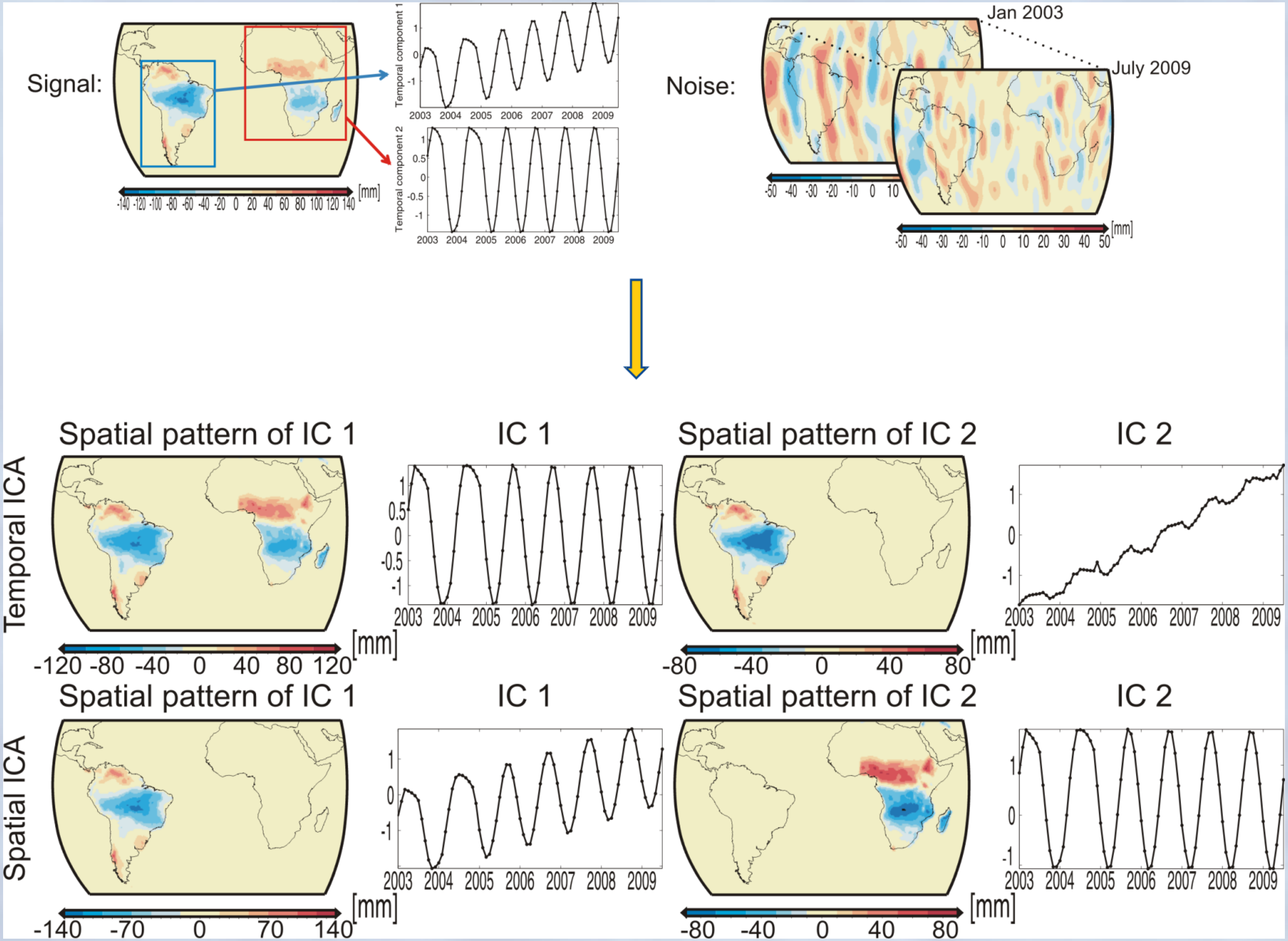
- Spatial ICA: Rotation of EOFs : $\mathrm{x} = \mathrm{E}_k\,\mathrm{R}$

$k \prec n,m$

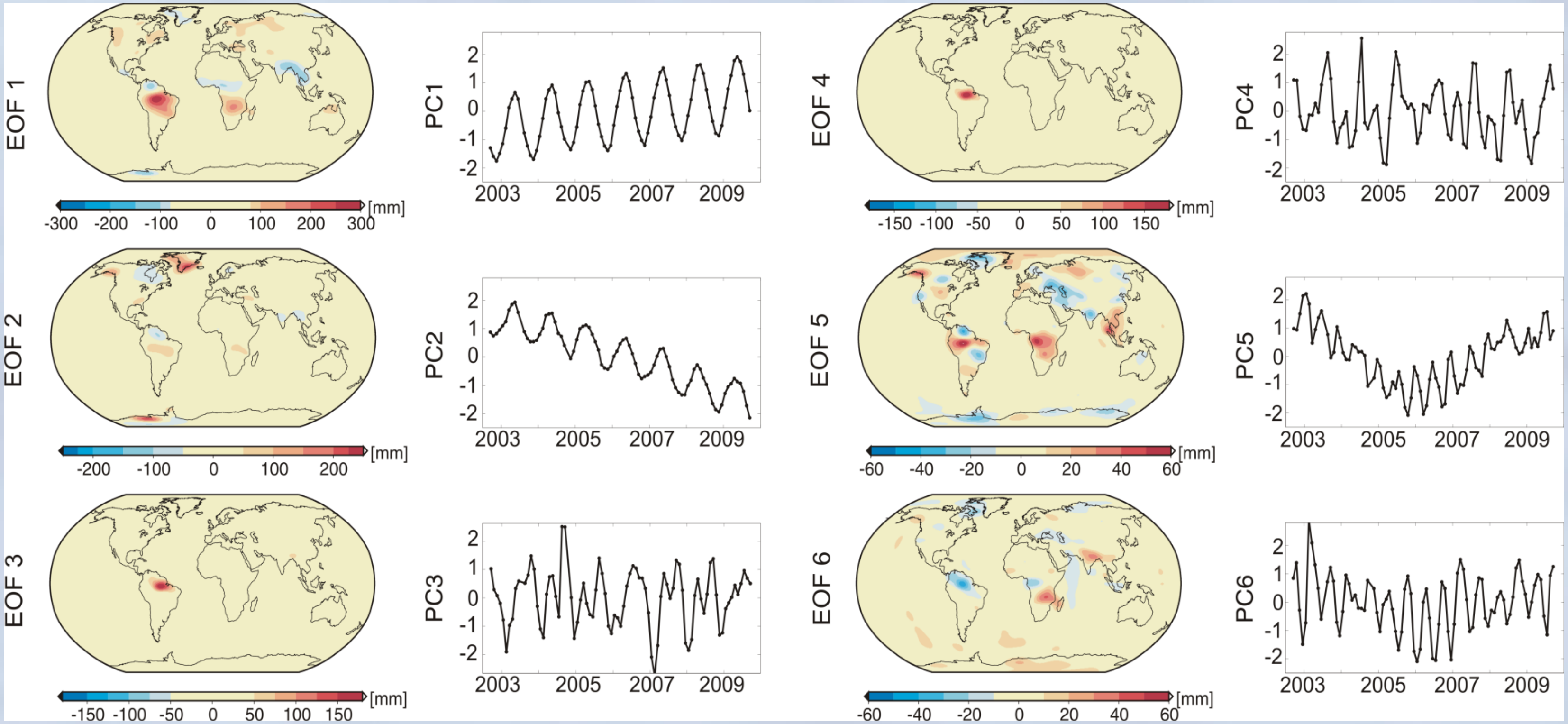
- Temporal ICA: Rotation of PCs : $\mathrm{x} = \mathrm{P}_k\mathrm{R}$

⟹ ICA criterion $\rightarrow f(\mathrm{x}) = Max\left(\sum_{j=1}^{k} C(x_j)^2\right)$ (Forootan and Kusche, submited)

ICA`s separation performance



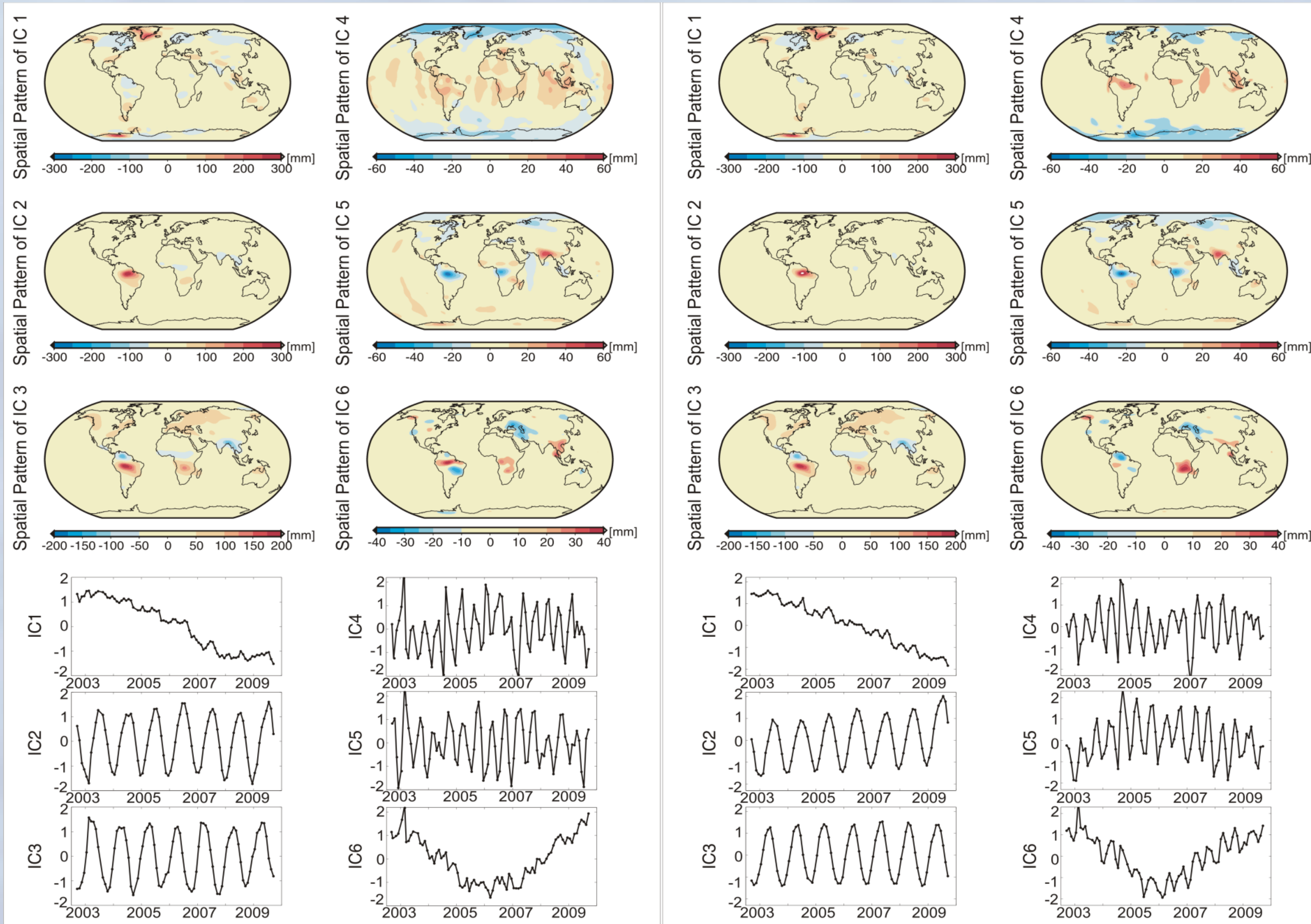
PCA decompositiom ITG2010



- PC1, PC2, PC3 and PC4 contain annual cycles.
- PC3 and PC4 are contaminated with semi-annual cycles.
- PC5 and PC6 contain semi-annual cycles.
- Spatial patterns are repeated in several components which makes the interpretation difficult.

(Data is pre-processed using Kusche, (2007)'s DDK2 filter)

Results of decomposing ITG2010



Temporal ICA

Spatial ICA

Results & Discussion

- PCA is suitable for dimension reduction.
- PCA's orthogonality constrain is restrictive for interpretation purpose.
- For non-Gaussian signals, ICA does what we want PCA to do for Gaussian Signal.
  - 2 steps ICA algorithm
    - PCA, as an initial step, improves both the computational and interpretability of the decomposition procedure.
    - Rotating the components towards independence.
- Using a simulation, ICA showed a better performance with compare to the ordinary PCA to separate non-Gaussian signals.
- Within the real case, we believe that ICA improved the PCA's performance.
- ICA is not able to separate high correlated physical components.
  - Those separations should be investigated using different approaches.

Thank you for your attention

- Main references:

1. E. Lorenz, 1956. Empirical Orthogonal Function and Statistical Weather Prediction, Tech. Rep. Science Report No. 1 Statistical Forecasting Project, MIT, Cambridge U.S.A.

2. J.-F. Cardoso, 1998. Blind Signal Separation: Statistical Principles, Proceedings of the IEEE DOI - 10.1109/5.720250 86 (10), 2009--2025, ISSN 0018-9219.

3. E.Forootan and J.Kusche, Separation of Global Time-variable Gravity Signals into Maximally Independent Components, submitted in J.Geodesy.