Method of averaging of saturated granular media

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Introduction

Microstructure and properties of porous-viscous media are well studied at low porosity[1-2]. However, at the critical porosity, equal to about 30-40%, the existing methods of averaging are no longer accurate. We developed and improved methods of averaging three-liquid heterogeneous media, suitable for research in the near critical region. This method are suitable for both periodic and for a randomly inhomogeneous microstructure.

Most of the really existing liquefied rock, which may be regarded as porous-viscous media (partially molten rock, boiling sands, mud, marshes, liquefied soils) are granular mediums in which the grains are surrounded by a low-viscosity liquid. In the supercritical region, there is some qualitative change in the microstructure of the mixture, which is manifested in the fact that the shear viscosity decreases abruptly. In this region, neighboring grain pairs are almost touching each other, but that touch is never complete. Full contact is impossible, due to a "lubricating" force, which approaches infinity as the distance between the spheres approaches zero. We propose that the lubricating layer can be described by considering it as an anisotropic viscous fluid, with symmetry properties aligned with the axis connecting the centers of contacting spheres. Pressure and viscous stresses in the (narrow) gaps take values much higher than away from them.

Averaging of a dispersed mixture consist of two stages. First, following [3], we average the motion of spheres with lubrication between them. The result is a two-phase heterogeneous mixture. The first phase of the mixture consists of an anisotropic fluid with high viscosity, while the second phase - from the isotropic liquid of low viscosity. Averaging leads to the Darcy's law and the defining equation for the skeletal phase. Where porosity is subcritical, granules are adjacent to each other, interacting in the way described above, and averaging leads to the local equations of compaction.

Three-liquid heterogeneous media consist of highly viscous spheres (I), they could be considered solid, lubricant layer (II), that directly joins neighboring spheres and low-viscous fluid (III), that fills all other volume between spheres. Fluids II and III are described by anisotropic rheological equations.

The first stage of averaging

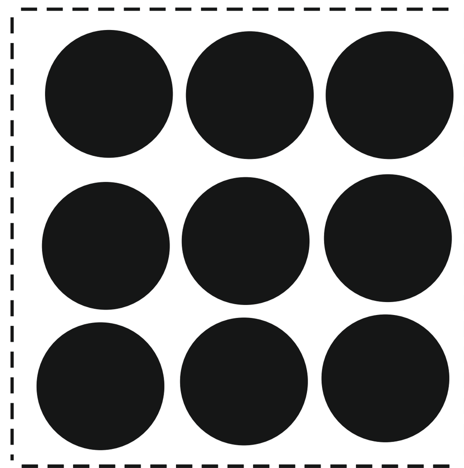
Stock's equation for anisotropic media contains viscosity tensor.

$$\hat{\sigma} = \hat{\tau} - p\hat{I}, \quad \hat{\tau} = \hat{\eta}\cdot\hat{e}, \quad \hat{e} = \left(\nabla\otimes\vec{w} + \left(\nabla\otimes\vec{w}\right)^T\right), \nabla\cdot\vec{w} = 0$$

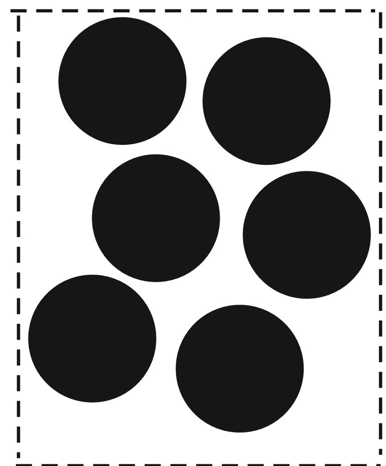
$$\frac{\partial}{\partial x_k}\eta^s_{iklm}\chi(\vec{x})\frac{\partial}{\partial x_l}w_m + \frac{\partial}{\partial x_k}\eta^f_{iklm}\left[1-\chi(\vec{x})\right]\frac{\partial}{\partial x_l}w_m = f_i(\vec{x})$$

$\eta^f_{iklm} = \eta(\delta_{il}\delta_{km} - \delta_{kl}\delta_{im})$, $\chi(\vec{x}) = 1$ while $\vec{x}\in\Omega^s$ and

$\chi(\vec{x}) = 0$ while $\vec{x}\in\Omega^f$. $\vec{f}$ is external force



At this figure scheme of periodical granular-medium is shown



At this figure scheme of non periodical structure is shown

Diagram technique

Green's function $G_{ik}(\vec{x},\vec{x}')$ is introduced. $w_i(\vec{x}) = \int d\vec{x}' G_{ik}(\vec{x},\vec{x}') f_k(\vec{x}')$.

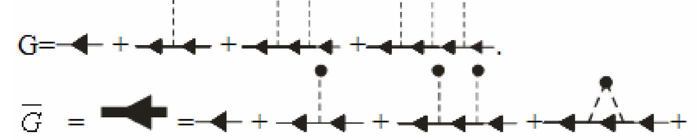
Green's function satisfies Lippmann-Schwinger integral equation

$G_{ik}(\vec{x},\vec{x}') = G^0_{ik}(\vec{x},\vec{x}') + \int d\vec{x}'' G^0_{ij}(\vec{x},\vec{x}'') V_{jl}(\vec{x}'') G_{lk}(\vec{x}'',\vec{x}')$, where G^0_{ik} – is 'free' Green's function

$$\frac{\partial^2}{\partial x_l^2}G^0_{ik}(\vec{x},\vec{x}') = -\delta_{ik}\delta(\vec{x}-\vec{x}'), \quad \frac{\partial}{\partial x_i}G^0_{ik}(\vec{x},\vec{x}') = 0, \quad V_{ik}(\vec{x}) = \frac{\partial}{\partial x_l}\eta_{iklm}\chi(\vec{x})\frac{\partial}{\partial x_m}, \eta_{iklm} = \eta^s_{iklm} - \eta^f_{iklm}.$$

Expanding into a row leads to $G = G^0 + G^0 * V * G^0 + G^0 * V * G^0 * V * G^0 + \ldots$

Graphical symbols are introduced: $G^0 \equiv$, $V \equiv$. That leads to



Where $\bar{V} \equiv$, $\overline{V'V'} \equiv$. $V' \equiv V - \bar{V}$, $\chi' = \chi - \bar{\chi}$.

Mass operator is introduced $\Sigma =$ 

Dyson equation is obtained $\bar{G} = G^0 + G^0 * \Sigma * \bar{G}$.

$$\bar{G}_{ik}(\vec{x}-\vec{x}') = G^0_{ik}(\vec{x}-\vec{x}') + \int d\vec{x}'' d\vec{x}''' G^0_{ij}(\vec{x}-\vec{x}'')\Sigma_{jl}(\vec{x}''-\vec{x}''')\bar{G}_{lk}(\vec{x}'''-\vec{x}')$$

$$\Sigma_{ik}(\vec{x}-\vec{x}') = \frac{\partial}{\partial x_l}\eta^{ef}_{iklm}(\vec{x}-\vec{x}')\frac{\partial}{\partial x'_m}.$$

Non local dependency of average distortion stress tensor and average distortion speed tensor is obtained

$$\bar{\tau}_{ik}(\vec{x}) = \int dx' \eta^{ef}_{iklm}(\vec{x}-\vec{x}')\frac{\partial w_m}{\partial x'_l}(\vec{x}').$$

Quasi-local approximation lead to

$$\bar{\tau}_{ik}(\vec{x}) = \int dx' \eta^{ef}_{iklm}(\vec{x}')\frac{\partial w_m}{\partial x'_l}(\vec{x}') + \int x'_p dx' \eta^{ef}_{iklm}(\vec{x}')\frac{\partial^2 w_m}{\partial x'_p \partial x'_l}(\vec{x}') +$$

$$+\frac{1}{2}\int x'_p x'_q dx' \eta^{ef}_{iklm}(\vec{x}')\frac{\partial^3 w_m}{\partial x'_p \partial x'_q \partial x'_l}(\vec{x}')$$

Integra-differential equation of compaction of statistically homogeneous medium is obtained

$$\frac{\partial}{\partial x_k}\int dx' \eta^{ef}_{iklm}(\vec{x}-\vec{x}')\frac{\partial w_m}{\partial x'_l}(\vec{x}') = f_i(\vec{x}).$$

While reaching rather high porosity processes of destruction and recovery of connections are commenced. While reaching critical value of porosity the number of broken connections is enlarging and percolation clusters of particles with broken connections are appearing. Dilantacy processes leads to destruction of connections. It due to non-consitence of particle positions. It leads to short-term dilation at certain points of media where destruction of connections takes place. Relative movement of particles may revoke this non-consistence and connections would reappear. In the end the condition of dynamic equilibrium appears between destruction and appearing of connections. This condition depends on porosity: changing of porosity leads to changing of equilibrium point. This condition is averaged by 2 methods: 1) using Feynman diagram technique and 2) averaging at phase volume .

H. Ziegler proposed[5] mechanism of movement at phase volume: mass is conserved in all phase volume, but not at elementary phase volume. It means that both convective and conductive transporting could be taken place at phase volume. Conductive transporting is similar to quasi-diffusion. Formally that leads to appearing at right part of Liouville's equation a part of a source. This source is a divergence of some vector. According to Ziegler's idea, movement at connections destroying would be equaled to the quasi-diffusion process. Virtual fluctuations at virtual continuum are introduced. Ziegler's idea allows us to use thermodynamics of irreversible processes to explain the process of connections destroying.

Conclusions

- Diagram technique methods were used to obtain exact averaged constitutive equations for three-saturated media.
- Justification of phenomenological local compaction equations was obtained.
- This technique allows to solve the equations at all fluid concentrations
- It is shown that with increasing of porosity non local terms occur, which correspond to the emergence of clusters.
- Note that this work is methodological. The most difficult and tedious aspects associated with the derivation of non-local constitutive equations were worked out. The analysis of those equations is beyond the scope of this work. For simplicity, we consider only the hydrodynamic interaction between particles in heterogeneous mixture. In reality, other types of interaction of the particles can exist, e.g. of a molecular nature. Physico-chemical communication significantly complicates the situation and makes all the calculations more cumbersome. However, they don't alter the fundamental line of reasoning.

References

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