



Expansion of the gravitational potential in triaxial ellipsoidal harmonics

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Spherical harmonics have been extensively used in geodesy because they are relatively simple and the shape of the earth is nearly spherical. However, since the shape of the earth is closer to an ellipsoid of revolution, spheroidal harmonics have also been used. In modern geodesy, the triaxial ellipsoid as a generalization of the ellipsoid of revolution will have a significant role to play in studying the figure of the earth. In the era of outer space explorations, small bodies of the solar system are becoming the target of current and forthcoming space missions. These bodies have irregular shapes and the triaxial ellipsoid, being a genuine three-dimensional shape, provides a very good approximation. Thus, it might be expected that ellipsoidal harmonics, which are defined in a way similar to that of the spheroidal harmonics, would be even more suitable for the representation of the gravitational field of the earth, asteroids and comets. The purpose of the presentation is to discuss the theory of ellipsoidal harmonics and the basic background required to solve Dirichlet's boundary-value problem for a triaxial ellipsoid. We introduce triaxial ellipsoidal coordinates and we express Laplace's equation in these coordinates. By applying the method of separation of variables to Laplace's equation, the solution is obtained by solving Lamé's differential equation. For this reason, we present Lamé's functions in some detail. Using these functions, we formulate the ellipsoidal harmonics expansion of the gravitational potential in the exterior of a triaxial ellipsoid. Also, we show that the spherical and spheroidal harmonics can be produced as degenerated cases of the ellipsoidal harmonics. In spite of the fact that ellipsoidal harmonics are more complicated than spherical or spheroidal harmonics, they can be used in certain special cases which nevertheless are important, such as in modeling, for instance, the gravity field of a level triaxial ellipsoid.