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Gravity modeling: the Jacobian function and its approximation

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In mathematics, the elements of a Jacobian matrix are the first-order partial derivatives of a scalar function or a vector function with respect to another vector. In inversion theory of geophysics the elements of a Jacobian matrix are a measure of the change of the output signal caused by a local perturbation of a parameter of a given (Earth) model.

The elements of a Jacobian matrix can be determined from the general Jacobian function. In gravity modeling this function consists of the “geometrical part” (related to the relative location in 3D of a field point with respect to the source element) and the “source-strength part” (related to the change of mass density of the source element).

The explicit (functional) expressions for the Jacobian function can be quite complicated and depend both on the coordinates used (Cartesian, spherical, ellipsoidal) and on the mathematical parametrization of the source (e.g. the homogenous rectangular prism). In practice, and irrespective of the exact expression for the Jacobian function, its value on a computer will always be rounded to a finite number of digits. In fact, in using the exact formulas such finite representation may cause numerical instabilities. If the Jacobian function is smooth enough, it is an advantage to approximate it by a simpler function, e.g. a piecewise-polynomial, which numerically is more robust than the exact formulas and which is more suitable for the subsequent integration.

In our contribution we include a whole family of the Jacobian functions which are associated with all the partial derivatives of the gravitational potential of order 0 to 2, i.e. including all the elements of the gravity gradient tensor. The quality of the support points for the subsequent polynomial approximation of the Jacobian function is ensured by using the exact prism formulas in quadruple precision. We will show some first results.

Also, we will discuss how such approximated Jacobian functions can be used for large scale modeling so that the source model can (almost) be of unlimited degree of detail and where the modeling can be conducted without a substantial increase in the computational task. The main idea here is to integrate instead of summing up the partial contributions from the discrete source elements.