



Exact Jacobians in an implicit Newton method for two-phase flow in porous media

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Geological storage of CO₂ is one option for mitigating the effects of CO₂ emissions on global warming. Since extensive on-site monitoring of the CO₂ plume propagation is expensive, numerical simulations are an attractive alternative for gaining deeper insight in the dynamics of this system. We consider a model for two-phase flow in porous media for representing the injection stage of a CO₂ sequestration scenario, when the plume propagation is dominated by advection.

The porous medium filled by the two phases CO₂ and brine is modelled as an initial-boundary-value problem consisting of two nonlinear, coupled partial differential equations, which are complemented by appropriate boundary and initial conditions. We present a new numerical approach to solve this fully coupled system using exact Jacobians. The method is based on the finite element, finite volume, box method [Huber & Helmig(2000)] for the space discretization and, since stability of the method is one of the main concerns, the fully implicit Euler method for the time discretization. A simple first order upwind method takes into account advective contributions. The resulting system of nonlinear algebraic equations is linearized by Newton's method. The required Jacobians can be obtained elegantly by automatic differentiation (AD) [Griewank & Walther(2008), Rall(1981)], a source code transformation giving exact derivatives of the discretized equations with respect to primary variables. The resulting system of linear equations is then solved by an iterative method (BiCGStab) with ILU0 preconditioning in every Newton step.

We compare the forward AD differentiation mode to the standard finite difference method in terms of precision and performance. It turns out that AD performs favourable in both aspects. We also illustrate the advantages of exact Jacobians for two-phase flow in a sequestration scenario investigating the evolution of pressure and saturation.

References

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