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Simple models of the earth's energy balance interpret some qualitative aspects of the dynamics of paleo-climatic data. They may be modeled by dynamical systems perturbed by noise, typically

$$X^\epsilon(t) = x - \int_0^t U'(X^\epsilon(s-))\, ds + \epsilon L(t), \quad t \geq 0, \epsilon > 0, \tag{1}$$

where L is a Lévy process. Its originally assumed Gaussian nature was questioned in the climate physics literature by Ditlevsen [1,2] who analyzed global temperature proxies in a Greenland ice-core time series from the last glacial period and suggested jump diffusion models with *α-stable noise* instead.

In statistical terms, Ditlevsen's conjecture leads to a typical model selection problem. In the parametric version involved, one needs an efficient testing method for instance for the parameter α corresponding to the best fitting α-stable noise component.

We access the non-Gaussian component by measuring the regularity of the trajectories in the model by *power variations*, and develop a statistical testing method for heavy tailed *Lévy noise* in the SDE. The power variation of order p, $V_t^p(f)$, of a function $f : [0, \infty) \to \mathbf{R}$ is defined as the limit of the functionals

$$V_t^{p,n}(f) = \sum_{i=1}^{[nt]} |f(\frac{i}{n}) - f(\frac{i-1}{n})|^p, \text{ namely } \quad V_t^p(f) = \lim_{n\to\infty} V_t^{p,n}(f), \quad t \geq 0\ .$$

It is apparent that $V^p(X^\epsilon)$ does not depend on the absolutely continuous part related to the potential gradient and that it is solely determined by the driving noise. Furthermore the choice of large values of p allows to damp out a possible Gaussian noise component. In the case of α-stable noise power variations are α/p-stable and Lévy themselves as a result of a functional limit theorem (see [3,5]).

The method compares the empirical variation paths with their theoretical limits in terms of the *Kolmogorov-Smirnov* (uniform) and *Wasserstein metrics*. We subdivide the data set into blocks and measure the power variation along these blocks. Then the distance between their empirical distribution and their α/p-stable limit law provides the test statistic. For further analysis the behavior of the test statistic is characterized by its convergence speed and its asymptotic distribution (see [4]).

In an application to the ice-core time series our method identifies the presence of a non-Gaussian component consistent with Ditlevsen's findings.

References

[1] P. D. Ditlevsen, *Anomalous jumping in a double-well potential*. Physical Review E **60(1)** (1999), 172–179.

[2] P. D. Ditlevsen, *Observation of α-stable noise induced millenial climate changes from an ice record*. Geophysical Research Letters **26(10)** (1999), 1441–1444.

[3] P. Imkeller, *Simple SDE dynamical models interpreting climate data and their meta-stability*. Mathematisches Forschungsinstitut Oberwolfach. Report Nr. **50**/2008.

[4] J. Gairing and P. Imkeller, *Dynamical models of climate time series and the rate of convergence of power variations*. Mathematisches Forschungsinstitut Oberwolfach. Report Nr. **40**/2011.

[5] C. Hein, P. Imkeller and I. Pavlyukevich, *Limit theorems for p-variations of solutions of SDEs driven by additive non-Gaussian stable Lévy noise*. Preprint, Humboldt Universität zu Berlin (2008).