



Non equilibrium statistical mechanics of geophysical flows

F. Bouchet

ENS-Lyon and CNRS, Lyon, France (Freddy.Bouchet@ens-lyon.fr)

Onsager first proposed to explain the self organization of turbulent flows using the statistical mechanics framework. Generalization of those ideas to the class of 2D-Euler and Quasi-Geostrophic models led to the Robert-Sommeria-Miller theory. This approach was successful in modeling many geophysical phenomena: the Great Red Spot of Jupiter [?, ?], drift of mesoscale ocean vortices [?, ?], self-organization of Quasi-Geostrophic dynamics in mid-basin jets similar to the Gulf-Stream and the Kuroshio [?, ?], and so on.

However, this type of equilibrium theories fail to take into account forces and dissipation. This is a strong limitation for many geophysical phenomena. Interestingly, it is possible to circumvent these difficulties using the most modern theoretical development of non-equilibrium statistical mechanics: large deviation [?] and instanton theories.

As an example, we will discuss geophysical turbulent flows which have more than one attractor (bistability or multistability). For instance, paths of the Kuroshio [?], the Earth's magnetic field reversal, atmospheric flows [?], MHD experiments [?], 2D turbulence experiments [?, ?], 3D flows [?] show this kind of behavior.

On Navier-Stokes and Quasi-Geostrophic turbulent flows, we predict the conditions for existence of rare transitions between attractors, and the dynamics of those transitions. We discuss how these results are probably connected to the long debated existence of multi-stability in the atmosphere and oceans, and how non-equilibrium statistical mechanics can allow to settle this issue.

Generalization of statistical mechanics to more comprehensive hydrodynamical models, which include gravity wave dynamics and allow for the possibility of energy transfer through wave motion, would be extremely interesting. Namely, both are essential in understanding energy balance of geophysical flows. However, due to difficulties in essential theoretical parts of the statistical mechanics approach, previous methods describing statistical equilibria were up to now limited to the use of quasi-geostrophic models. We will discuss a recent study that fills this gap [?]. The new theory we propose describes geophysical phenomena using statistical mechanics applied to the shallow water model, and is easily generalizable to the primitive equations. Invariant measures of the shallow water model are built based on the Hamiltonian structure and the Liouville theorem. We discuss applications to the selection of vortex profiles.

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