



A stochastic model for temperature variability and climate sensitivity ranging from months to tens-of-thousands of years

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The spectral power of surface temperature variability scales with frequency, approximately following power laws of the form $P(\omega) \sim \omega^{-\beta}$. Several regimes can be identified in spectral estimates drawn from both instrumental and proxy records that span monthly to ten-thousand-year timescales. At frequencies above the seasonal cycle the spectrum exhibits red noise behavior ($\beta \sim 2$); between annual and centennial frequencies it is shallower ($0 < \beta < 1$); and at lower frequencies it again steepens ($1 < \beta < 2$). Previous studies have described these power law scalings in both models and observations, and have proposed mechanisms for particular scalings such as stochastic resonance, turbulent transport in the atmosphere, and $1/f$ diffusion scaling in the ocean. However, these studies focus on single power-law processes without accounting for the existence of several regimes and the transitions between them.

Here we propose a physically based model that captures the various scaling regimes through a representation of ocean-atmosphere interactions at higher frequencies and cryosphere-ocean/atmosphere interactions at lower frequencies. More specifically, we posit an energy balance model containing a shallow mixed layer, a diffusively coupled deep ocean, fast atmospheric feedbacks, and a slow feedback attributable to ice-sheet albedo or CO_2 . When driven by uncorrelated stochastic forcing the model produces a power-law scaling consistent with observations between monthly and ten-thousand-year timescales, which can be understood as resulting from different amplification of radiative perturbations by frequency-dependent feedbacks. An analytical solution based on the system's transfer function is presented in the Fourier domain that exactly describes the origin of these frequency-dependent feedbacks and resulting regimes of power-law scaling.

This transfer function constitutes itself in a time scale-dependent climate sensitivity. The frequency structure of the model's sensitivity to radiative perturbations indicates that the magnitude of the response depends critically upon the timescale of the forcing, which is distinct from the behavior of the fast-feedbacks usually considered within climate sensitivity studies. For instance, when the model is fit to instrumental and paleoclimate records, we find that, for the same forcing, the response at ten-thousand-year timescales is approximately 4 times larger in magnitude than would be a decadal response. Thus, by constraining the model parameters to fit observational estimates for the background continuum scaling, empirical bounds can be placed on the magnitude of climate sensitivity at different time scales.