



Energy transfer in geostrophic turbulence at finite Rossby and Froude number

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A deep understanding of turbulent processes mainly focused on how energy is exchanged among the different scales is crucial, both from a scientific and a practical point of view. Critical evaluations as well as related improvements of large-scale atmospheric models cannot be achieved unless the physics and the main mechanism which the atmosphere rely on are understood.

We hypothesize that the observed wave number spectra of kinetic and potential energy in the atmosphere can be explained by assuming that there are two related cascade processes emanating from the same large scale energy source, a downscale cascade of potential enstrophy (half the square of potential vorticity), giving rise to the k^{-3} -spectrum at synoptic scales and a downscale energy cascade giving rise to the $k^{-5/3}$ -spectrum at mesoscales. We also hypothesize that the amount of energy which is going into the downscale energy cascade is determined by the rate of system rotation, with negligible energy going downscale in the limit of very fast rotation. To test these hypotheses we carry out a set of very high resolution simulations of the primitive equations and the Boussinesq equations, with strong stratification, strong rotation and large scale forcing. We find that the amount of energy which is going into the downscale energy cascade decreases monotonically with increased rate of rotation and show that the downscale energy cascade generates a transition in the wave number spectrum, from $\sim k^{-3}$ to $\sim k^{-5/3}$, consistent with observations. We also show that the transition between the two dynamic regimes is associated with a change of sign of the third order structure function of velocity differences, consistent with observations from the lower stratosphere. Some results pertaining to the primitive equation were presented in an article recently published in *Physical Review Letters*¹.

In addition to these results we also present analyses of the transfer of energy between geostrophic and ageostrophic modes. Within the $k^{-5/3}$ -range, we find that there is approximate equipartition of energy between geostrophic and ageostrophic modes, consistent with the results of a stratified turbulent dynamics. In order to investigate the role of gravity waves, frequency analyses of time series of several Fourier modes are carried out, clearly showing the presence of large-scale wave motions. However, no distinct signature can be found at smaller scales. In order to investigate whether turbulent motions or wave resonance are mainly responsible for the energy transfer, analyses of triad interactions are carried out. We find that turbulent motions are dominant.

¹A. Vallgren, E. Deusebio, and E. Lindborg. A possible explanation of the atmospheric kinetic and potential energy spectra. *Phys. Rev. Lett.*, 99:99–101, December 2011.